EA

Lezione

derivate

27/11/19

DERIVATE - CALCOLO

$y = \sin x - \cos x$

$D(f(x) \pm g(x)) =$
$= f'(x) \pm g'(x)$

$y' = \cos x - (-\operatorname{sen} x) =$
$= \cos x + \operatorname{sen} x$

$D(\cos x) = -\operatorname{sen} x$

$D(\operatorname{sen} x) = \cos x$

$y = \operatorname{sen}(x^3 - 2x) - \cos(e^x)$

$D(f(g(x))) = f'(g(x)) \cdot g'(x)$

$D(x^n) = n\,x^{n-1}$

$D(e^x) = e^x$

$$y' = \cos(x^3 - 2x) \cdot \underbrace{(3x^2 - 2)}_{D(x^3-2x)} - \left[-\operatorname{sen}(e^x) \cdot e^x\right] =$$

$$= \cos(x^3 - 2x)(3x^2 - 2) + \operatorname{sen}(e^x)\, e^x$$

$$y = \sqrt{x^2 - 3x + 2} =$$

$$= (x^2 - 3x + 2)^{1/2}$$

$$-1/2 = \frac{1}{2} - 1$$

$$y' = \frac{1}{2}(x^2 - 3x + 2)^{-1/2} \cdot \underbrace{(2x - 3)}_{D(x^2-3x+2)}$$

$$y = \sqrt[3]{x^2 - 3x + 2} = (x^2 - 3x + 2)^{1/3}$$

$$y' = \frac{1}{3}(x^2 - 3x + 2)^{-2/3}(2x - 3)$$

$$y' = \frac{1}{2\sqrt{x^2-3x+2}}\,(2x-3)$$

$$y' = \frac{1}{3\sqrt[3]{(x^2-3x+2)^2}}\,(2x-3)$$

$$D\left(\sqrt{x^n}\right) = \frac{1}{2\sqrt{x^n}} \cdot \underbrace{n\,x^{n-1}}_{D(x^n)}$$

$$y = \sqrt{x \underset{\uparrow \text{ prodotto}}{\cdot} \log x}$$

$$D\left[f(x) \cdot g(x)\right] =$$

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$y' = \underbrace{\frac{1}{2\sqrt{x \cdot \log x}}}_{D\sqrt{\ }} \cdot \underbrace{\left(1 \cdot \log x + \cancel{x} \cdot \frac{1}{\cancel{x}}\right)}_{D \text{ Radicando}} =$$

$$D(\log x) = \frac{1}{x}$$

$$= \frac{\log x + 1}{2\sqrt{x \cdot \log x}}$$

$a > 0 \quad a \neq 1$

$$D(a^x) = a^x \ln a$$

$$D\left(a^{f(x)}\right) = a^{f(x)} \cdot \ln a \cdot f'(x)$$

$$D(\ln x) = \frac{1}{x}$$

$$D(\log_a x) = \frac{1}{x} \cdot \log_a e$$

$$y = \frac{x^2+5x-3}{3^x}$$

$D(K) = 0$

$$D\left[\frac{N(x)}{D(x)}\right] = \frac{N'(x)\cdot D(x) - D'(x)\cdot N(x)}{D^2(x)}$$

$$y' = \frac{(2x+5)\,3^x - \underbrace{3^x\cdot \ln 3}_{D(3^x)'}\cdot(x^2+5x-3)}{(3^x)^2} =$$

$$= \frac{\cancel{3^x}\left[2x+5-\ln 3\cdot(x^2+5x-3)\right]}{(3^x)^{\cancel{2}}} =$$

$$= \frac{\overset{a}{-\ln 3}\,x^2 + x\overset{b}{(2-5\ln 3)} + \overset{c}{5+3\ln 3}}{3^x}$$

$$y = (x^2 - 3)^{\operatorname{sen} 2x}$$

$$D\left[(f(x))^{g(x)}\right] =$$

$$(f(x))^{g(x)} = e^{\log (f(x))^{g(x)}} = e^{g(x) \cdot \log(f(x))}$$

$$D\left[e^{g(x) \cdot \log(f(x))}\right] =$$

$$= \underbrace{e^{g(x) \cdot \log(f(x))}}_{\text{se stesse}} \underbrace{\left[g'(x) \cdot \log(f(x)) + \frac{g(x)}{f(x)} \cdot f'(x)\right]}_{\text{D esponente (prodotto)}} =$$

$$= [f(x)]^{g(x)} \cdot \left[g'(x) \log(f(x)) + \frac{g(x)}{f(x)} \cdot f'(x) \right]$$

$$y = (x^2 - 3)^{\operatorname{sen} 2x} = e^{\log (x^2-3)^{\operatorname{sen} 2x}}$$

$$= e^{\operatorname{sen} 2x \cdot \log(x^2-3)} =$$

$$= e^{\operatorname{sen} 2x \cdot \log(x^2-3)}$$

$$y' = e^{\operatorname{sen} 2x \cdot \log(x^2-3)} \cdot \left(\underbrace{\underbrace{\cos(2x)}_{D \operatorname{sen}(2x)} \cdot \underbrace{2}_{D(2x)}} \cdot \log(x^2-3) + \frac{\operatorname{sen} 2x \cdot}{\underbrace{x^2-3}_{D \log(x^2-3)}} \cdot \underbrace{(2x)}_{D(x^2-3)} \right)$$

$$= (x^2-3)^{\operatorname{sen} 2x} \left(2 \cos 2x \, \log(x^2-3) + \frac{\operatorname{sen} 2x}{x^2-3} \cdot 2x \right)$$

• $y = \log(e^x + 1) + \operatorname{arctg} x$

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

$$D\{\operatorname{arctg}(f(x))\} = \frac{1}{1+f^2(x)} \cdot f'(x)$$

$$y' = \frac{1}{e^x+1} \cdot \underbrace{e^x}_{D(\text{arg. log})} + \frac{1}{1+x^2}$$

$$y = \log\left(e^{x^2-1} + 2x\right) + \operatorname{arctg}(3x^2 - 4x)$$

$$y' = \frac{1}{e^{x^2-1}+2x} \cdot \underbrace{\left(e^{x^2-1} \cdot \underset{D(x^2-1)}{2x} + 2\right)}_{D\text{ arg. log}} + \underbrace{\frac{1 \cdot \overset{D(3x^2-4x)}{(6x-4)}}{1+(3x^2-4x)^2}}_{D\text{ arctg}}$$

$$\cdot\ y = e^{\frac{3x^2+\sqrt{x}+1}{x^3}} \cdot \sqrt[4]{x^4+x^3+5} =$$

$$= e^{\frac{3x^2+\sqrt{x}+1}{x^3}} \cdot \left(x^4+x^3+5\right)^{1/4}$$

$$D\left(\frac{3x^2+\sqrt{x}+1}{x^3}\right) =$$

$$= \frac{\overbrace{\left(6x+\frac{1}{2\sqrt{x}}\right)}^{D\ Num} \cdot x^{\cancel{3}} - \overbrace{3\,\cancel{x^2}}^{D\ Den}\left(3x^2+\sqrt{x}+1\right)}{x^{\cancel{6}\ 4}} =$$

$$= \frac{\left(6x+\frac{1}{2\sqrt{x}}\right) \cdot x - 3\left(3x^2+\sqrt{x}+1\right)}{x^4} =$$

$$= \frac{\frac{\left(12x\sqrt{x}+1\right)}{2\sqrt{x}} \cdot \sqrt{x} - 9x^2 - 3\sqrt{x} - 3}{x^4} =$$

$$= \frac{12x^2+\sqrt{x} - 18x^2 - 6\sqrt{x} - 6}{2x^4} =$$

$$= \frac{-6x^2 - 5\sqrt{x} - 6}{2x^4}$$

$$D\left(x^4 + x^3 + 5\right)^{1/4} =$$

$$= \frac{1}{4}\left(x^4 + x^3 + 5\right)^{-3/4} \cdot \left(4x^3 + 3x^2\right)$$

D (esponente e)

$$y' = \underbrace{e^{\frac{3x^2+\sqrt{x}+1}{x^3}}}_{} \cdot \overbrace{\frac{-6x^2 - 5\sqrt{x} - 6}{2x^4}} \cdot$$

$$\cdot \sqrt[4]{x^4 + x^3 + 5} \quad +$$

$$+ \underbrace{e^{\frac{3x^2+\sqrt{x}+1}{x^3}}}_{} \cdot \frac{1}{4}\left(x^4 + x^3 + 5\right)^{-3/4} \left(4x^3 + 3x^2\right)$$

• T.E.

$$f(x) = \frac{5x}{\sqrt{x^2+1}} + 2\sqrt{x^2-3x+3}$$

DOM: $x^2-3x+3 \geq 0$

$$x_{1,2} = \frac{3 \pm \sqrt{9-12}}{2}$$

$\Delta < 0$ no sol.

verificata $\forall x \in \mathbb{R}$

$$y' = \frac{5\left(\sqrt{x^2+1}\right) - \frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x \cdot 5x}{(x^2+1)} +$$

$$+ \cancel{2} \frac{1}{\cancel{2}\sqrt{x^2-3x+3}} \cdot (2x-3) =$$

$$= \frac{5(x^2+1) - 5x^2}{(x^2+1) \cdot \sqrt{x^2+1}} + \frac{2x-3}{\sqrt{x^2-3x+3}} =$$

$$= \frac{5}{(x^2+1)\sqrt{x^2+1}} + \frac{2x-3}{\sqrt{x^2-3x+3}}$$

• $y = \log(1+e^{2x}) + \sqrt{x^2-4x+5}$

Dom: $1+e^{2x} > 0 \quad \forall x \in \mathbb{R}$

$x^2-4x+5 \geq 0 \qquad x_{1,2} = \dfrac{4 \pm \sqrt{16-20}}{2}$ $\Delta<0$

$\forall x \in \mathbb{R}$

$f(x) \geq 0$

$$\underbrace{\log(1+e^{2x})}_{\text{segno}} + \underbrace{\sqrt{x^2-4x+5}}_{+} \geq 0$$

$\log(1+e^{2x}) > 0$ se

$1+e^{2x} > 1$

$e^{2x} > 0 \qquad \forall x \in \mathbb{R}$

$\Rightarrow f(x)$ è la somma di due quantità positive

$$y' = \frac{1}{1+e^{2x}} \cdot \underbrace{e^{2x}\cdot 2}_{\text{D arg. log}} + \frac{1}{2\sqrt{x^2-4x+5}} \cdot (2x-4) =$$

$$= \frac{2e^{2x}}{1+e^{2x}} + \frac{2x-4}{2\sqrt{x^2-4x+5}}$$

$$\cdot\ y = \log\left(e^{|x|} - 1\right) + 2e^{-x} + 2$$

N.B.

$$y = |x| = \begin{cases} x & \text{per } x \geq 0 \\ -x & \text{per } x < 0 \end{cases}$$

$$y' = D(|x|) = \begin{cases} 1 & \text{per } x \geq 0 \\ -1 & \text{per } x < 0 \end{cases} =$$

$$= \operatorname{sgn}(x)$$ Funzione segno di x

quando devo derivare un modulo

- divido la funzione nelle sue due parti
- derivare tenendo conto del $\operatorname{sgn}(\text{Arg.} |\ |)$

$$y' = \frac{1}{e^{|x|}-1} \cdot \underbrace{e^{|x|}}_{D(\arg \log)} \cdot \overbrace{\mathrm{sgn}(x)}^{D(|x|)} + 2e^{-x} \cdot (-1) =$$

$$= \frac{e^{|x|}}{e^{|x|}-1} \cdot \mathrm{sgn}(x) - \frac{2}{e^{x}}$$

Scrivendo la funzione a tratti

$$y = \begin{cases} \log(e^{x}-1) + 2e^{-x} + 2 & \text{per } x \geq 0 \\ \log(e^{-x}-1) + 2e^{-x} + 2 & \text{per } x < 0 \end{cases}$$

$$y' = \begin{cases} \dfrac{e^{x}}{e^{x}-1} + 2 \cdot e^{-x}(-1) & \text{per } x \geq 0 \\ -\dfrac{e^{-x}}{e^{-x}-1} + 2\, e^{-x}(-1) & \text{per } x < 0 \end{cases}$$

stesso intervallo

- Verificare che la funzione

$$y = \frac{x^2 + 2x + 2}{2}$$

soddisfa l'equazione

$$1 + (y')^2 = 2yy''$$

$y \Rightarrow$ funzione

$y' \Rightarrow$ derivata prima della funzione

$y'' \Rightarrow$ derivata seconda della funzione

$$y = \frac{x^2 + 2x + 2}{2}$$

$$y' = \frac{2x + 2}{2}$$

$$y'' = \frac{2}{2} = 1$$

$$1 + (y')^2 = 2\,y\,y''$$

si sostituiscono i valori per la verifica

$$1 + \left(\frac{2x+2}{2}\right)^2 \overset{?}{=} 2 \cdot \frac{x^2+2x+2}{2} \cdot 1$$

$$1 + (x+1)^2 = x^2+2x+2 \quad ?$$

$$1 + x^2+2x+1 = x^2+2x+2 \quad ?$$

$$x^2+2x+2 = x^2+2x+2$$

Sì

T.E. 2015

$$f(x) = \begin{cases} \dfrac{x^2}{x+2} + \sqrt{x}\,\sin\dfrac{1}{x} & \text{se } x > 0 \\ 0 & \text{se } x = 0 \\ x \ln|x| & \text{altrimenti } (x < 0) \end{cases}$$

- f è continua in $x = 0$
- $y = x - 3$ Asintoto obliquo $x \to +\infty$ (destra)
- $f'(-2) = \ln 2 + 1$

? f continua in $x=0$?

$f(0) = 0$ per definizione

$$\lim_{x\to 0^+} f(x) \overset{?}{=} \lim_{x\to 0^-} f(x) \overset{?}{=} f(0)$$

$$\lim_{x\to 0^+} \frac{x^2}{x+2} + \sqrt{x} \cdot \sin\frac{1}{x} =$$

$$= \frac{0}{2} + \sqrt{0^+} \cdot [-1, 1] = (???)$$

$$= 0$$

$$\lim_{x\to 0^-} x \ln|x| = 0$$

si è continua in $x=0$

? asintoto obliquo $x \to +\infty$?

$$\lim_{x\to+\infty} \left[\frac{x^2}{x+2} + \sqrt{x}\ \text{sen}\ \frac{1}{x}\right] = +\infty + 0 = $$
$$= +\infty$$

$$+\infty \leftarrow \sim x \leftarrow \frac{x^2}{x+2} \simeq \frac{x^2}{x} \simeq x$$

$$\sqrt{x} \cdot \text{sen}\ \frac{1}{x}$$

$$+\infty \cdot 0$$

$$\lim_{x\to+\infty} \sqrt{x} \cdot \text{sen}\ \frac{1}{x} =$$

$$= \lim_{x\to+\infty} \sqrt{x} \cdot \frac{\text{sen}\ \frac{1}{x}}{x \cdot \frac{1}{x}} \simeq \lim_{x\to+\infty} \frac{\sqrt{x}}{x} = 0$$

$$\frac{\text{sen}\ 0}{0} \to 1$$

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} =$$

$$= \lim_{x\to+\infty} \left(\frac{x^2}{x+2} + \sqrt{x}\cdot \operatorname{sen}\frac{1}{x}\right)\cdot\frac{1}{x} =$$

$$= \lim_{x\to+\infty} \left\{\frac{x^2}{x(x+2)} + \frac{\sqrt{x}}{x}\cdot \operatorname{sen}\frac{1}{x}\right\} =$$

$$= \lim_{x\to+\infty} \left\{\underbrace{\frac{x^2}{x^2\left(1+\frac{2}{x}\right)}}_{1} + \frac{1}{\sqrt{x}}\operatorname{sen}\frac{1}{x}\right\} = 1$$

$$\underbrace{\frac{1}{\sqrt{x}}\cdot\frac{\operatorname{sen}\frac{1}{x}}{x\cdot\frac{1}{x}}}_{0} \qquad \frac{\operatorname{sen}\frac{1}{x}}{\frac{1}{x}} \to 1$$

$$q = \lim_{x \to +\infty} \left(f(x) - \underset{1}{m} x \right) =$$

$$= \lim_{x \to +\infty} \left(\frac{x^2}{x+2} + \sqrt{x} \sin \frac{1}{x} - x \right) =$$

$$= \lim_{x \to +\infty} \left(\frac{\cancel{x^2} - \cancel{x^2} - 2x}{x+2} + \sqrt{x} \sin \frac{1}{x} \right) =$$

$$= \lim_{x \to +\infty} \left(\underbrace{\frac{-2x}{x+2}}_{\downarrow \, -2} + \sqrt{x} \sin \frac{1}{x} \right) = -2$$

$$\sqrt{x} \cdot \frac{\sin \frac{1}{x}}{x \cdot \frac{1}{x}} \qquad \frac{\sin \frac{1}{x}}{\frac{1}{x}} \to 1$$

$$0 \leftarrow \frac{\sqrt{x}}{x}$$

$$? \ f'(-2) = \ln 2 + 1 \ ?$$

$x = -2$ $\qquad\qquad\qquad\qquad$ $\underline{x < 0}$

$$y = x \ln |x| =$$

$$= x \ln (-x)$$

$$y' = 1 \cdot \ln(-x) + x \cdot \frac{1}{-x} \cdot (-1) =$$

$D(\ln \ldots)$ $\qquad$ $D(-x)$

$$= \ln(-x) + 1$$

$$f'(-2) = \ln(+2) + 1 \quad \text{OK}$$

↑ coeff. angolare della retta
Tangente per $x = -2$
per $f(x) = x \ln(-x)$

CALCOLO DELLA RETTA TANGENTE ALLA FUNZIONE IN $x = -2$

Eq. retta passante per un punto

$\cancel{y=}$ $P(x_p, y_p)$

m

$$y - y_p = \underbrace{m}_{f'(x_p)}(x - x_p)$$

$$y_p = f(x_p) =$$

$$y = f'(x_p)(x - x_p) + f(x_p)$$

valore della derivata prima in P

valore della funzione in P

- $f'(-2) = \ln 2 + 1$

- $y = x \ln|x| = x \ln(-x)$

$f(-2) = -2 \ln 2$ ordinata del punto P.

Retta Tangente in $P(-2; -2\ln 2)$

$$y = [\ln 2 + 1](x+2) + (-2\ln 2) =$$

$$= (\ln 2 + 1)(x+2) - 2\ln 2$$

$$y = \operatorname{arctg} x$$

$$D(\operatorname{arctg} x) = \frac{1}{1+x^2}$$

$$D(\operatorname{arctg} f(x)) = \frac{1}{1+(f(x))^2} \cdot f'(x)$$

$$y = \operatorname{arctg} x \rightarrow y' = \frac{1}{1+x^2}$$

$$y = \operatorname{arctg}^2\left(\frac{x+1}{x-1}\right)$$

$$y' = \underbrace{2 \cdot \operatorname{arctg}\left(\frac{x+1}{x-1}\right)}_{D(t^2)} \cdot \underbrace{\frac{1}{1+\left(\frac{x+1}{x-1}\right)^2}}_{D \operatorname{arctg}} \cdot \underbrace{\frac{x-1-x-1}{(x-1)^2}}_{D\left(\frac{x+1}{x-1}\right)} =$$

$$= 2 \operatorname{arctg}\left(\frac{x+1}{x-1}\right) \cdot \frac{\cancel{(x-1)^2}}{(x-1)^2 + (x+1)^2} \cdot \frac{-2}{\cancel{(x-1)^2}} =$$

$$= -4 \operatorname{arctg}\left(\frac{x+1}{x-1}\right) \cdot \frac{1}{2x^2+2} =$$

$$= -\frac{2}{x^2+1} \operatorname{arctg}\left(\frac{x+1}{x-1}\right)$$

$$y = \operatorname{arctg} |x+1| =$$

$$= \begin{cases} \operatorname{arctg}(x+1) & x+1 \geq 0 \quad x \geq -1 \\ \operatorname{arctg}(-x-1) & x+1 < 0 \quad x < -1 \end{cases}$$

$$y' = \begin{cases} \dfrac{1}{1+(x+1)^2} \cdot \overset{D(x+1)}{(1)} = \dfrac{1}{x^2+2x+2} & x \geq -1 \\[2ex] \dfrac{1}{1+(-x-1)^2} \cdot \overset{D(-x-1)}{(-1)} = \dfrac{-1}{x^2+2x+2} & x < -1 \end{cases}$$

$$y = \operatorname{arctg} |f(x)|$$

$$y' = \frac{1}{1 + \underbrace{|f(x)|^2}_{f(x)^2}} \cdot \operatorname{Sgm}[f(x)]$$

FUNZIONE SEGNO

$$y = \operatorname{sgn}(f(x)) = \begin{cases} 1 & f(x) > 0 \\ -1 & f(x) < 0 \end{cases}$$

$$y = x \cdot \log|x^2-1|$$

$$y' = 1 \cdot \log|x^2-1| + x \frac{1}{|x^2-1|} \cdot 2x \cdot \operatorname{sgn}(x^2-1)$$

D log $D(x^2-1)$ D|…|

se devo fare lo studio del segno, pr. non derivabilità devo sempre esprimerla nelle ~~qu~~ 2 parti

$$y' = \begin{cases} - - - & x - . \\ - - & x < \end{cases}$$

$$y = \operatorname{arctg} x + \frac{1}{x} - \frac{3}{x^3}$$

$$x \neq 0$$

$$y = \operatorname{arctg} x + x^{-1} - 3\,x^{-3}$$

$$y' = \frac{1}{1+x^2} + (-1)\,x^{-2} - 3(-3)\,x^{-4}$$

$$= \frac{1}{1+x^2} - \frac{1}{x^2} + \frac{9}{x^4}$$

$$y = \frac{1}{x^n} = x^{-n}$$

$$y' = -n\,x^{-n-1} = \frac{-n}{x^{n+1}}$$

Derivate

esempi di derivate di alcune funzioni elementari

$D\,k = 0$	$D\ 5 = 0$	$D\ \pi = 0$	$D\ 0 = 0$	$D\ \log_2 5 = 0$
$D\,x^n = n\,x^{n-1}$	$D\,x = 1$	$D\,x^7 = 7\,x^6$	$D\,x^{-2} = -2\,x^{-3}$	$D\ x^{\frac{2}{3}} = \frac{2}{3}\,x^{-\frac{1}{3}}$
$D\sqrt[n]{x} = \dfrac{1}{n\sqrt[n]{x^{n-1}}}$	$D\ \sqrt[3]{x} = \dfrac{1}{3\sqrt[3]{x^{3-1}}} = \dfrac{1}{3\sqrt[3]{x^2}}$		$D\ \sqrt[8]{x} = \dfrac{1}{8\sqrt[8]{x^{8-1}}} = \dfrac{1}{8\sqrt[8]{x^7}}$	
$D\,\log_a x = \dfrac{1}{x}\log_a e$	$D\,\log_3 x = \frac{1}{x}\log_3 e = \dfrac{1}{x}\,\dfrac{1}{\ln 3}$		$D\ \log_{\frac{1}{5}} x = \frac{1}{x}\log_{\frac{1}{5}} e = \dfrac{1}{x}\,\dfrac{1}{\ln\frac{1}{5}}$	
$D\,a^x = a^x \ln a$	$D\ 2^x = 2^x \ln 2$		$D\ 2012^x = 2012^x \ln 2012$	

esempi di derivate con le regole di derivazione

Derivata del prodotto di una costante k per una funzione $D\,k \cdot f(x) = k \cdot f'(x)$

$D\,(5\,x^2) = 5 \cdot D\,(x^2) = 5 \cdot (2\,x) = 10\,x$

$D\left(\frac{7}{3}\,senx\right) = \frac{7}{3} \cdot D(\,senx\,) = \frac{7}{3} \cdot cosx$

Derivata della somma di due o più funzioni $D\,f(x) \pm g(x) \pm h(x) = f'(x) \pm g'(x) \pm h'(x)$

$D(\,7\,\log_{10} x + 3x + 4\,) = \frac{1}{x} \cdot \frac{7}{\ln 10} + 3$

$D(\,5x^3 - tgx + x\,) = 15\,x^2 - \frac{1}{cos^2 x} + 1$

Derivata del prodotto di due funzioni $D\,f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$D\,(\,x^2\,tgx\,) = 2x \cdot tgx + x^2 \cdot \frac{1}{cos^2 x}$

$D\,(\,7\ln x \cdot e^x\,) = 7\,\frac{1}{x} \cdot e^x + 7\ln x \cdot e^x = 7e^x\left(\frac{1}{x} + \ln x\right)$

Derivata del rapporto di due funzioni $D\,\frac{f(x)}{g(x)} = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

$$D\left(\frac{2+x}{3x}\right) = \frac{(1) \cdot 3x - (2+x) \cdot 3}{(3x)^2} = \frac{3x - 3(2+x)}{(3x)^2} = \frac{x - 2 - x}{3x^2} = -\frac{2}{3\,x^2}$$

Derivata di una funzione composta $Df[g(x)] = f'[g(x)] \cdot g'(x)$

$D(\sqrt{senx}) = \frac{1}{2\sqrt{senx}} \cdot cosx$

$D(sen\sqrt{x}) = cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$D\,(sen^2 x\,) = (2\,senx) \cdot cosx = 2\,senx\,cosx$

$D\,(\,sen\,x^2\,) = (\cos x^2) \cdot 2x = 2\,x\cos x^2$

Derivata di una funzione elevata ad una funzione $Df(x)^{g(x)} = f(x)^{g(x)} \cdot \left[g'(x) \cdot \ln f(x) + g(x) \cdot \frac{f'(x)}{f(x)}\right]$

$D\,(\,x^{cosx}\,) = x^{cosx} \cdot \left[-senx \cdot \ln(x) + cosx \cdot \frac{1}{x}\right]$

$D\,(\,(senx)^x\,) = (senx)^x \cdot \left[1 \cdot \ln(senx) + x \cdot \frac{cosx}{senx}\right]$

FORMULARIO: tavola delle derivate fondamentali

$$y = f(x) \Rightarrow y' = f'(x)$$

FUNZIONE COSTANTE: $y = c \Rightarrow y' = 0$

FUNZIONE POTENZA: $y = x^n$ con $n \in \mathbb{R} \Rightarrow y' = nx^{n-1}$

$y = x \Rightarrow y' = 1$

$y = \dfrac{1}{x} \Rightarrow y' = -\dfrac{1}{x^2}$

$y = \sqrt{x} \Rightarrow y' = \dfrac{1}{2\sqrt{x}}$

$y = \sqrt[n]{x^m} \Rightarrow y' = \dfrac{m}{n\sqrt[n]{x^{n-m}}}$

FUNZIONE VALORE ASSOLUTO: $y = |x| \Rightarrow y' = \dfrac{x}{|x|}$

FUNZIONE LOGARITMICA: $y = \log_a x \Rightarrow y' = \dfrac{1}{x}\log_a e$

$y = \ln x \Rightarrow y' = \dfrac{1}{x}$

FUNZIONE ESPONENZIALE: $y = a^x \Rightarrow y = a^x \ln a$

$y = e^x \Rightarrow y' = e^x$

FUNZIONI GONIOMETRICHE	FUNZIONI GONIOMETRICHE INVERSE
$y = senx \Rightarrow y' = \cos x$	$y = arcsenx \Rightarrow y' = \dfrac{1}{\sqrt{1-x^2}}$
$y = \cos x \Rightarrow y = -senx$	$y = \arccos x \Rightarrow y' = -\dfrac{1}{\sqrt{1-x^2}}$
$y = tgx \Rightarrow y' = \dfrac{1}{\cos^2 x} = 1 + tg^2 x$	$y = arctgx \Rightarrow y' = \dfrac{1}{1+x^2}$
$y = ctgx \Rightarrow y' = -\dfrac{1}{sen^2 x}$	$y = arcctgx \Rightarrow y' = -\dfrac{1}{1+x^2}$

REGOLE DI DERIVAZIONE:

derivata di una **somma** di funzioni: $D(k \cdot f(x) + h \cdot g(x)) = k \cdot f'(x) + h \cdot g'(x)$

derivata di un **prodotto**: $D(f(x) \cdot g(x)) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

derivata di un **rapporto**: $D\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$

derivata di una **funzione composta** (funzione di funzione):

$$D(g(f(x))) = g'(f(x)) \cdot f'(x)$$