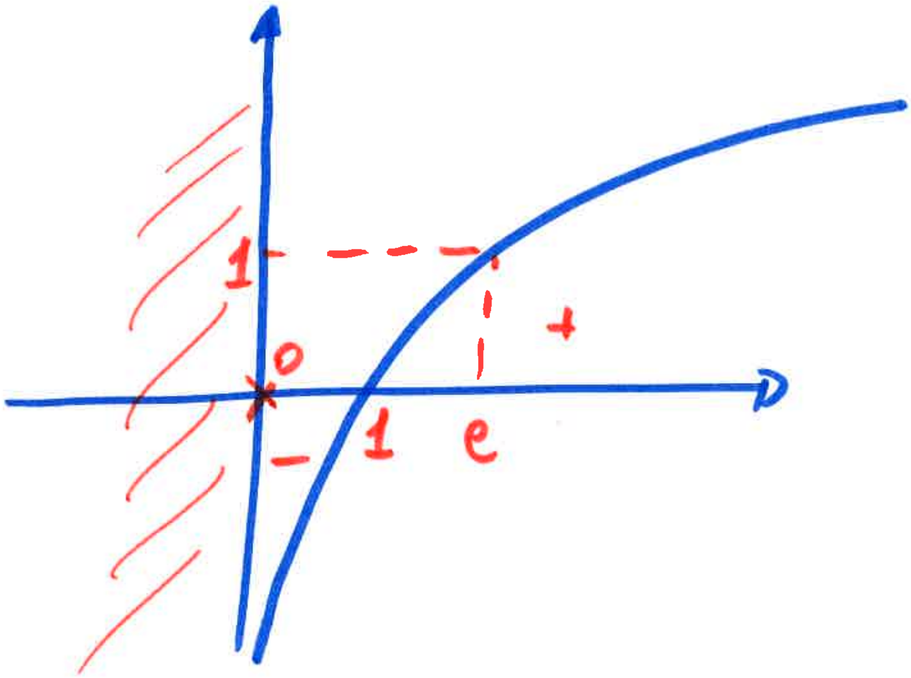
E. A

Lezione

Limiti funzioni reali

21/11/19

SIGNIFICATO GEOMETRICO DI LIMITE



$y = \ln x$

$A =]0, +\infty[$

$$\lim_{x \to 0^+} \ln x = -\infty$$

$\downarrow$ $]0, \delta[$ parte destra

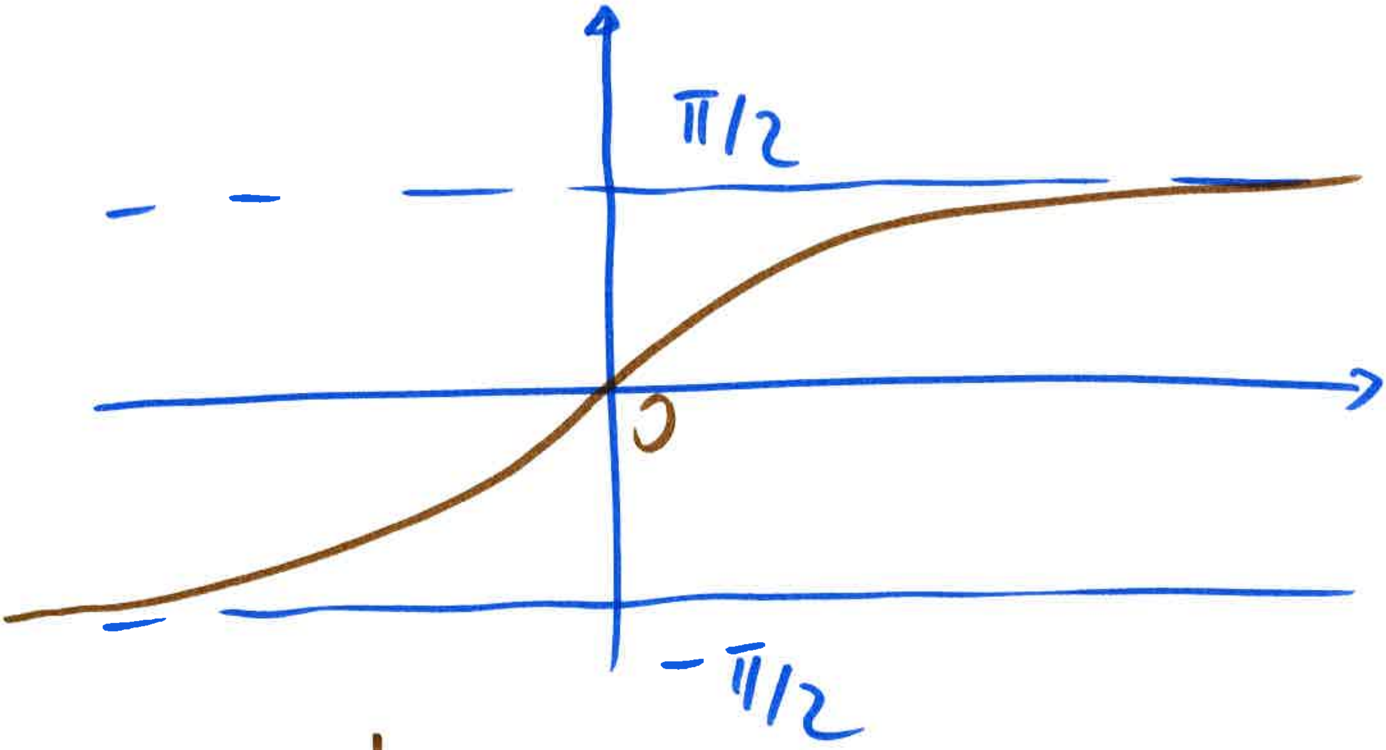
$x = 0$ pt. di acc. per A

$X = X_0$ ASINTOTO VERTICALE

A è superiormente illimitato

$$\lim_{x \to +\infty} \ln x = +\infty$$

NON ESISTE $\lim_{x \to 0^-} f(x)$ / $\lim_{x \to -\infty} f(x)$



$y = \operatorname{arctg} x$

$$\lim_{x \to 0} \operatorname{arctg} x = 0 \qquad (x, f(x))$$

$A =]-\infty, +\infty[$ inferiormente e superiormente illimitato

$$\lim_{x \to +\infty} \operatorname{arctg} x = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \operatorname{arctg} x = -\pi/2$$

$y = \pi/2$; $y = -\pi/2$ ASINTOTI ORIZZONTALI

ESISTENZA E UNICITA'

$$\lim_{x \to +\infty} \cos x = \nexists \qquad [-1, 1]$$

$A =]-\infty, +\infty[$

$-1 \le \cos x \le 1$

$y=-1 \qquad -1 \le \lim_{x \to +\infty} \cos x \le 1 \qquad y=1$

$$\lim_{x \to +\infty} [x + 5\cos x] = +\infty$$

$-1 \le \cos x \le 1$

$-5 \le 5 \cdot \cos x \le 5$

$x - 5 \le x + 5\cos x \le x + 5$

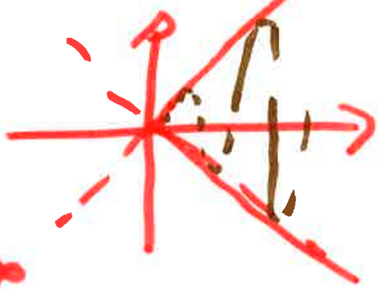
$\downarrow +\infty \qquad\qquad \downarrow +\infty$

$$\lim_{x \to +\infty} \frac{3}{2}\pi\, x \cdot \cos x = \nexists$$

$-1 \le \cos x \le 1$

$-x \le x\cos x \le x$

$\downarrow -\infty \qquad \downarrow +\infty$



$$\lim_{x \to +\infty} \sqrt{1 - x^2} = \nexists$$

$A = [-1, 1]$

$1 - x^2 \ge 0$

$$y = \sqrt{1-x^2}$$

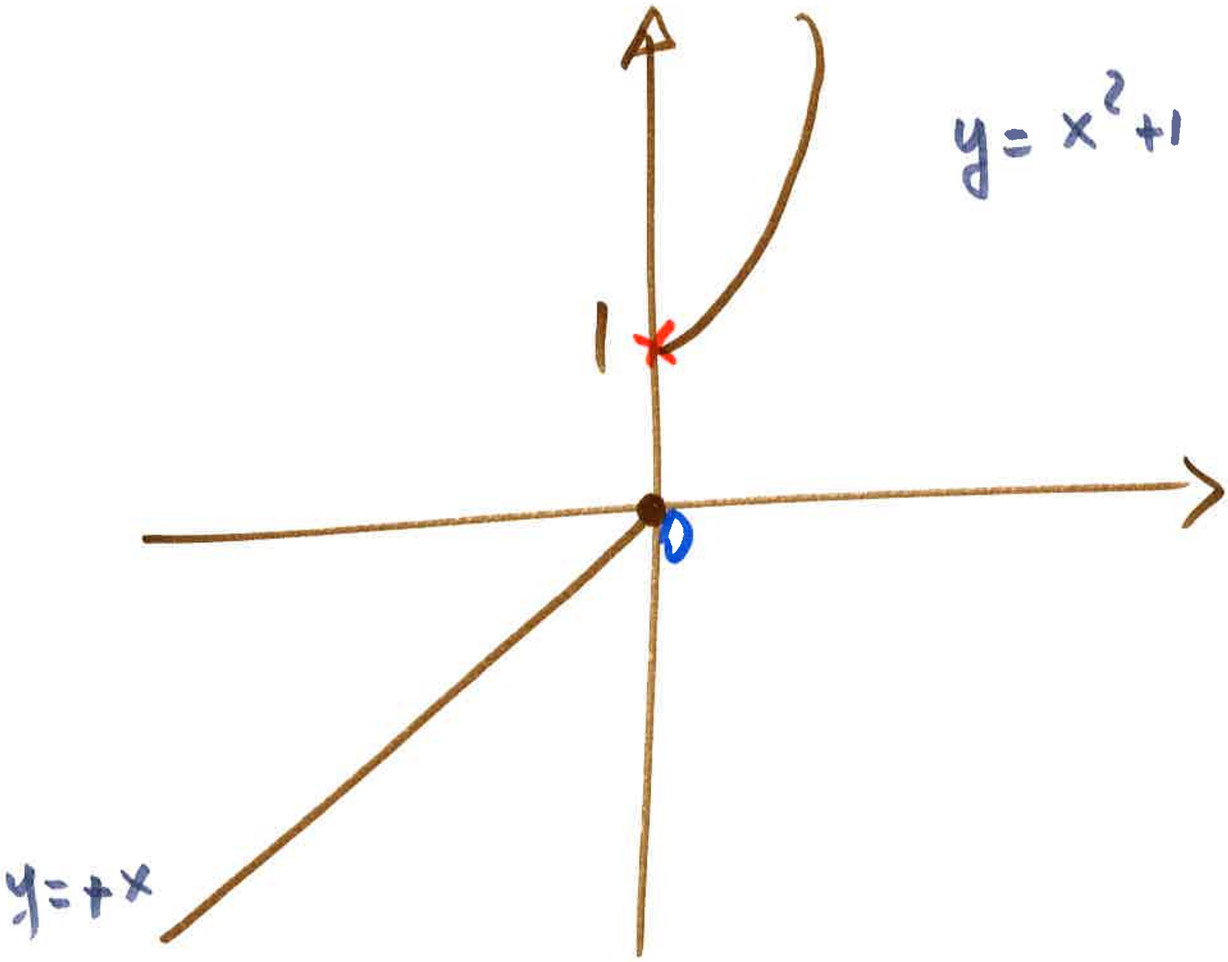
$$\lim_{x \to +\infty} \sqrt{(1-x^2)} = \nexists$$

$A = [-1, 1]$

limitato

dom: $1 - x^2 \geq 0$

$-1 \leq x \leq 1$



$$\lim_{x \to 0} f(x) = \nexists$$

Intorno completo

$$\lim_{x \to 0^-} f(x) = 0$$

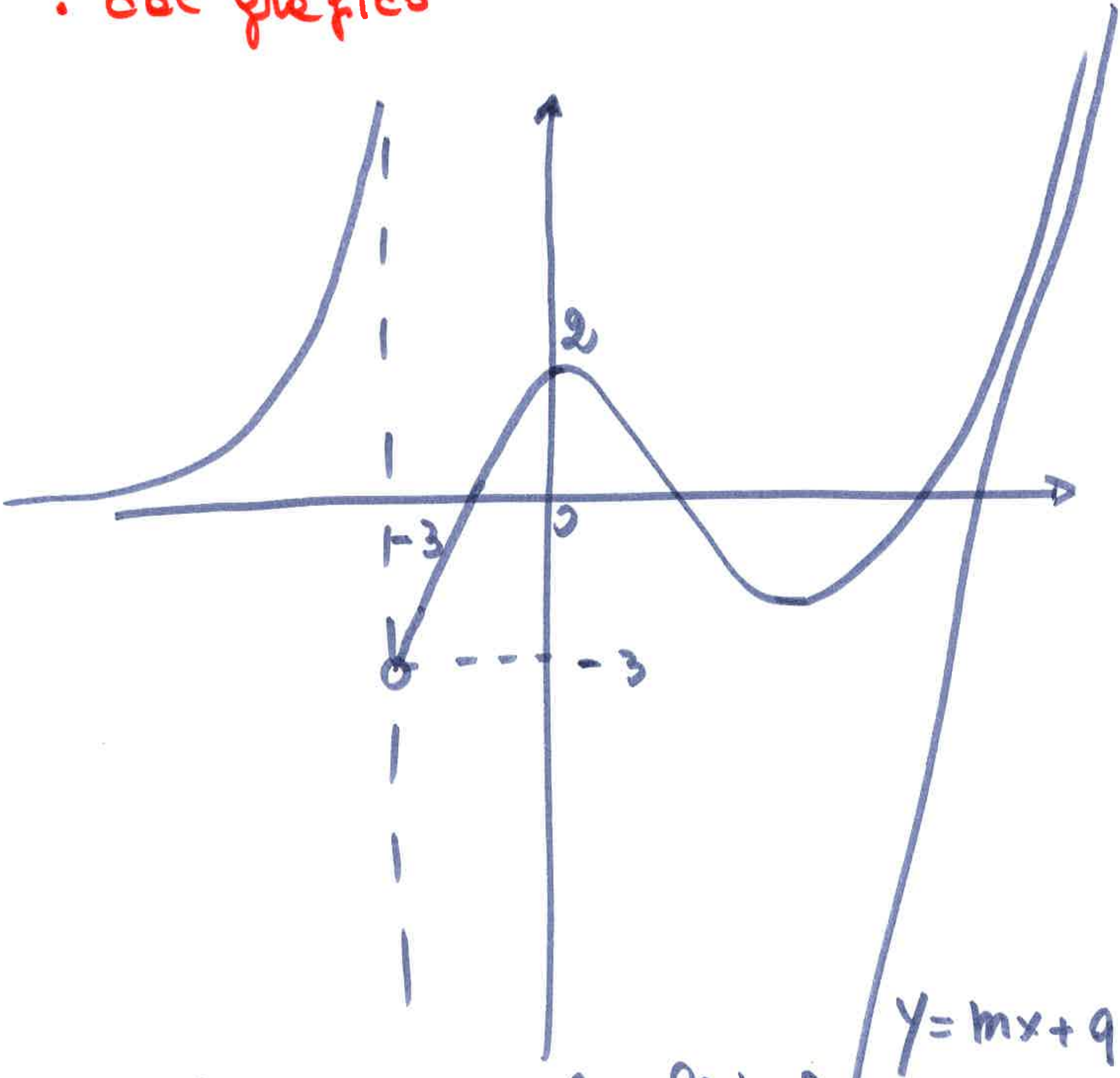
$$\lim_{x \to 0^+} f(x) = 1$$

SALTO

NON COINCIDONO

SIGNIFICATO DI LIMITE

- dal grafico



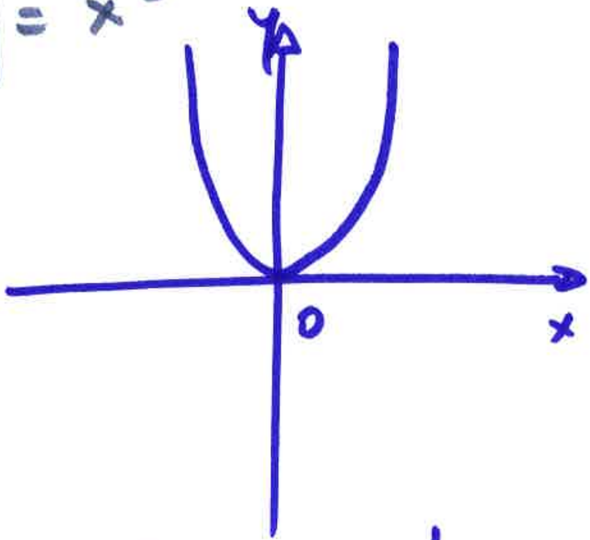
$$\lim_{x \to -\infty} f(x) = 0 \qquad \lim_{x \to 0} f(x) = 2$$

$$\lim_{x \to -3^-} f(x) = +\infty \qquad \lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -3^+} f(x) = -3$$

LIMITI FUNZIONI ELEMENTARI

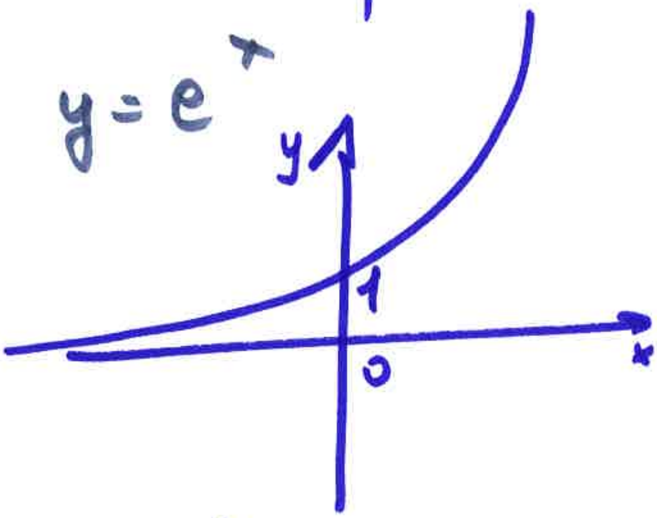
$y = x^2$



$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = +\infty$$

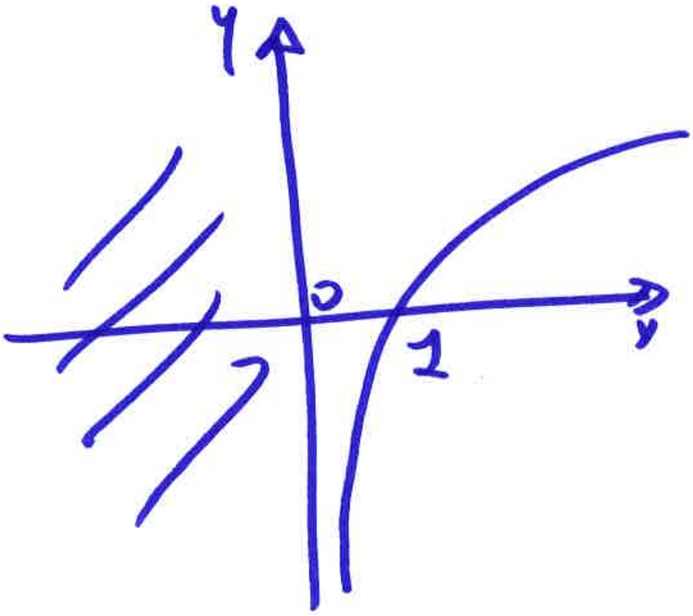
$y = e^x$



$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = 0$$

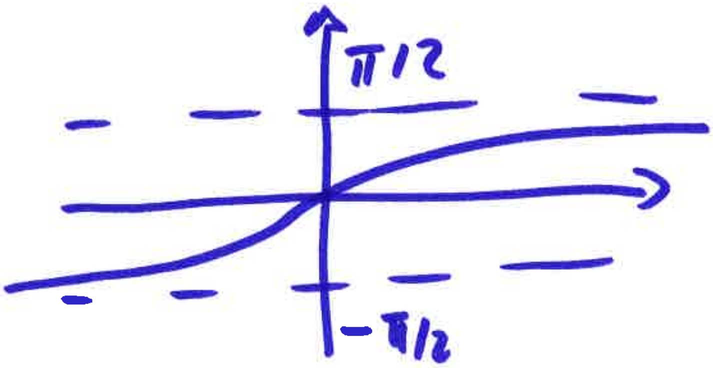
$y = \log x$



$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to 0^+} f(x) = -\infty$$

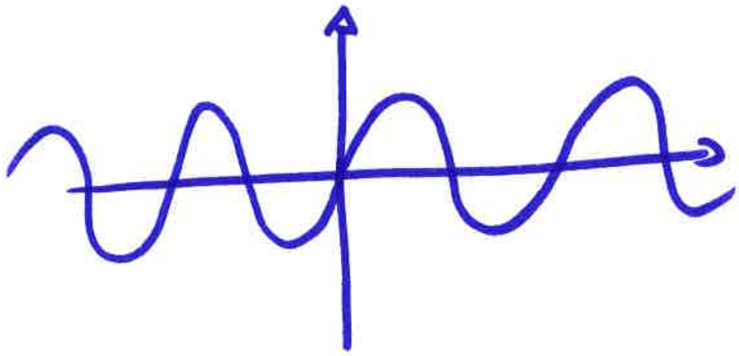
$y = \operatorname{arctg} x$



$\lim_{x \to +\infty} f(x) = +\frac{\pi}{2}$

$\lim_{x \to -\infty} f(x) = -\frac{\pi}{2}$

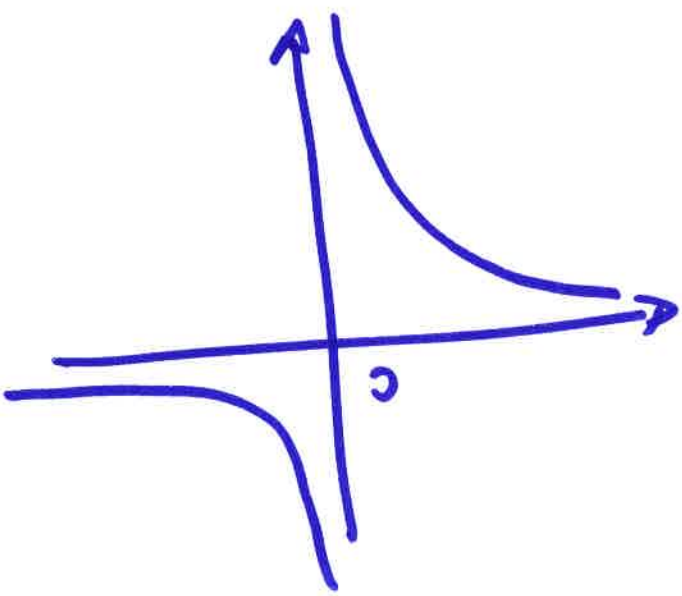
$y = \operatorname{sen} x$



$\lim_{x \to +\infty} f(x) = \nexists$

$\lim_{x \to -\infty} f(x) = \nexists$

$y = \frac{1}{x}$



$\lim_{x \to +\infty} f(x) = 0$

$\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 0^-} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = 0$

$$y = \frac{2x+3}{3x-1}$$

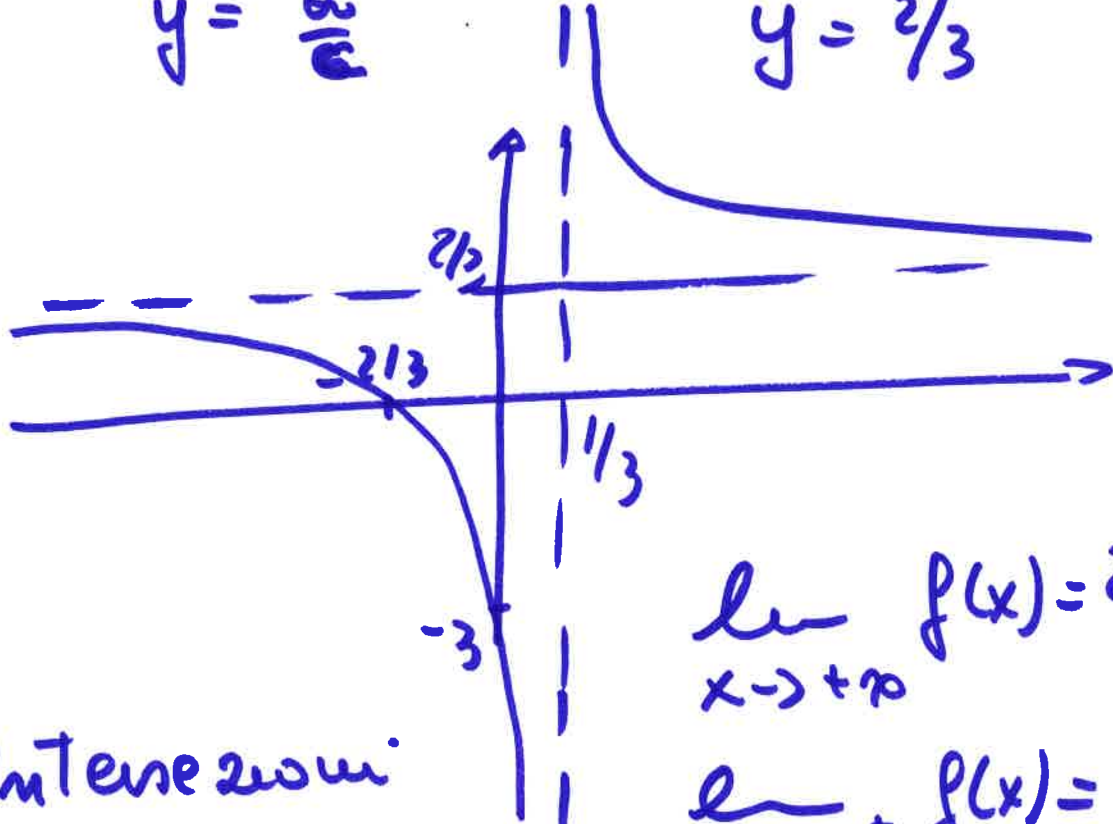
Funzione omografica

$$y = \frac{ax+b}{cx+d}$$

asintoti

$x = -\frac{d}{c}$ $\qquad$ $x = \frac{1}{3}$

$y = \frac{a}{c}$ $\qquad$ $y = 2/3$



$$\lim_{x \to +\infty} f(x) = 2/3$$

$$\lim_{x \to 1/3^+} f(x) = +\infty$$

$$\lim_{x \to \frac{1}{3}^-} f(x) = -\infty$$

$$\lim_{x \to -\infty} f(x) = 2/3$$

Intersezioni

$$\begin{cases} x = 0 \\ y = -3 \end{cases}$$

$$\begin{cases} y = 0 \\ x = -2/3 \end{cases}$$

Algebrici

$$\lim_{x \to 3} \frac{7x^2 - 2x}{3x + 1} =$$

Calcolare $\Rightarrow$ sostituire $x = 3$

$$= \frac{7 \cdot 9 - 2 \cdot 3}{3 \cdot 3 + 1} =$$

$$= \frac{63 - 6}{9 + 1} = \frac{57}{10}$$

FORME DI INDETERMINAZIONE

$+\infty - \infty$

$0\,(+\infty)$

$0\,(-\infty)$

$\frac{0}{0}$ $\quad$ $\frac{\infty}{\infty}$

$(\infty)^0$

1^∞

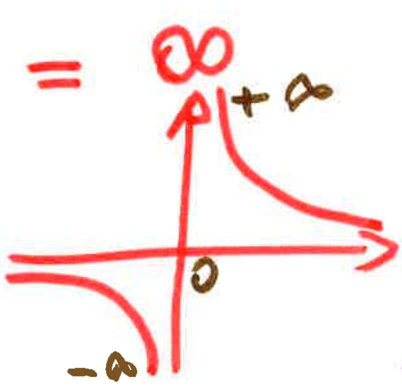
0^0

RICORDARSI

$\frac{K}{\infty} = 0$

$\frac{K}{0} = \infty$

$y = \frac{1}{x}$



RICORDARSI

LIMITI

- FORME DI INDETERM.
- CONFRONTO chi è più veloce?
- ORDINE DI INFINITO
- ORDINE DI INFINITESIMO
- LIMITI NOTEVOLI
- RACCOGLIMENTO FORZATO
- COMPORTAMENTO ASINTOTICO

applicazioni

- continuità
- nel grafico
- asintoti
  - verticale
  - orizzontale
  - obliquo

★ LE 7 FORME DI INDETERMINAZIONE

$$\left[+\infty - \infty\right] \quad \left[0 \cdot \infty\right] \quad \left[\frac{0}{0}\right]$$

$$\left[\frac{\infty}{\infty}\right] \quad \left[0^0\right] \quad \left[1^\infty\right] \quad \left[\infty^0\right]$$

★ NON SONO FORME DI INDETERMIN.

$$0^{+\infty} = 0^+$$

$$0^{-\infty} = \frac{1}{0^{+\infty}} = +\infty$$

$$\frac{\infty}{0^-} = \frac{L}{0^-} = \infty$$

$$\frac{\infty}{0^+} = \frac{L}{0^+} = \infty$$

INFINITI PREVALENTI

PIÙ VELOCI

e^x, a^x

x^n

PIÙ LENTI

$\ln x$, $\log_a x$ $\quad \underline{a>1}$

- se elevo a potenza, non viene modificata la prevalenza
- attenzione quando l'argomento di due infiniti non è lo stesso

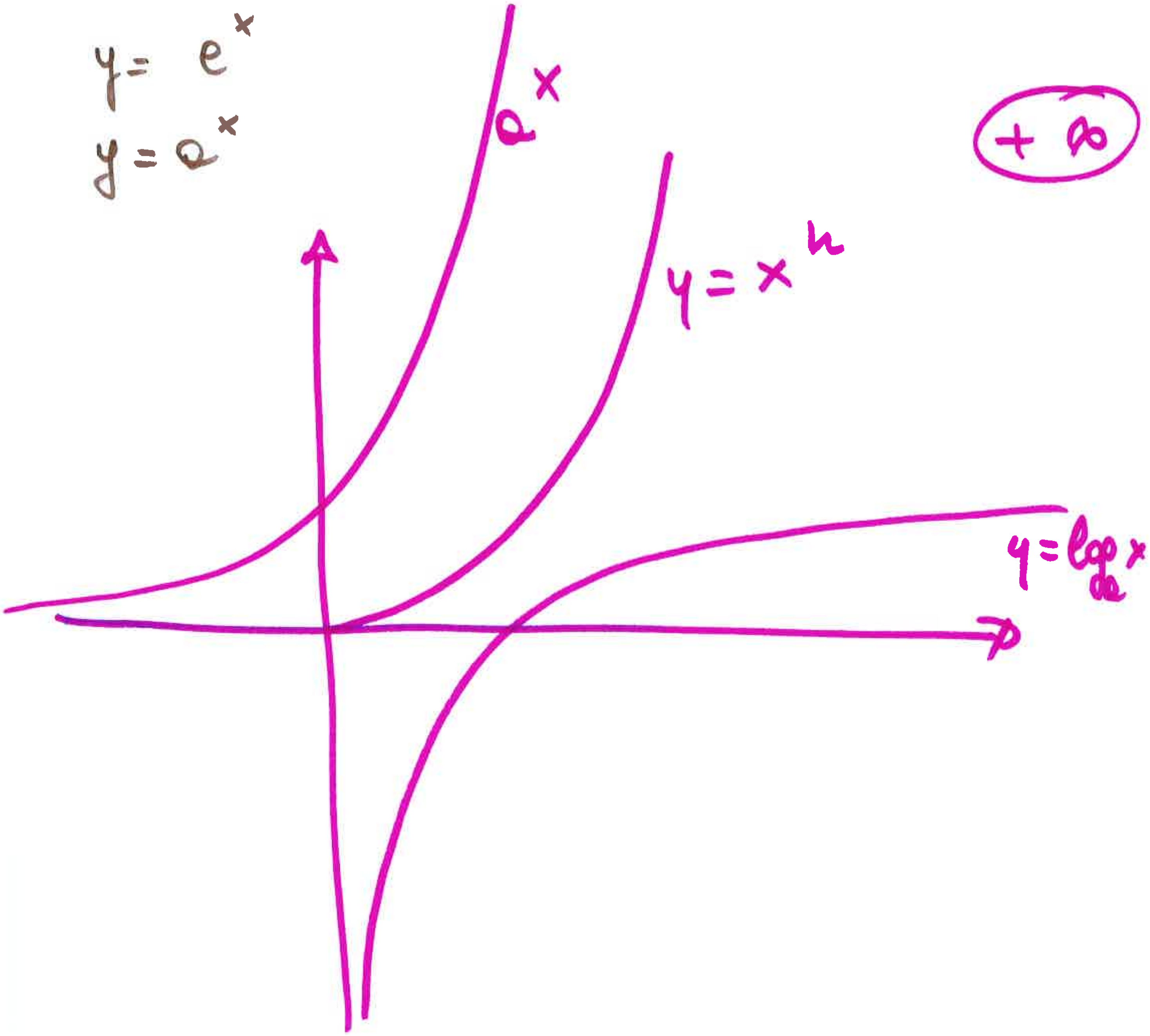
$y = \ln x = \log_e x$

$y = \log_e x \qquad a > 1$

$y = x^n = (P(x)) \qquad [y = \sqrt{x} = x^{\frac{1}{2}}]$

$y = e^x$

$y = a^x$



$$\lim_{x \to +\infty} \frac{e^x}{x^5} = +\infty$$

prevale e^x su x^5

$$\lim_{x \to +\infty} \frac{x^5}{e^x} = 0$$

prevale il den. sul num.

CONFRONTO TRA INFINITI

$$\frac{+\infty}{+\infty}$$

Nelle somme algebriche raccolgo ciò che va più velocemente a infinito.

$$\lim_{x \to +\infty} \frac{5\underline{e^x} + 4x^2}{\underline{e^{2x}} + \log x} =$$

Raccolgo è più veloce!

$$= \lim_{x\to+\infty} \frac{e^x\left(5+4\frac{x^2}{e^x}\right)}{e^{2x}\left(1+\frac{\log x}{e^{2x}}\right)} \simeq$$

$$\frac{x^2}{e^x} \rightsquigarrow 0$$

$$\frac{\log x}{e^{2x}} \rightsquigarrow 0$$

$$\simeq \lim_{x\to+\infty} \frac{5e^x}{e^{2x}} =$$

$$= \lim_{x\to+\infty} 5\, e^{x-2x} =$$

$$= \lim_{x\to+\infty} 5\, e^{-x} =$$

$$= 5\, e^{-\infty} = 0$$

★ IL SEGNO VIENE STABILITO

$$\frac{+\infty}{0^+} = +\infty$$

$$\frac{+\infty}{0^-} = -\infty$$

$$\frac{-1}{0^+} = -\infty$$

$$\frac{-1}{0^-} = -\infty$$

★ FUNZIONI RAZIONALI

- SI RAZIONALIZZA SOLO IN PRESENZA DI UNA FORMA INDETERMINATA

⇒

FORMA INDETERMINATA

⇓

- RAZIONALIZZARE
- POSSIBILITA' DI TRASCURARE I TERMINI

FORMA DETERMINATA

- POSSIBILITA' DI TRASCURARE I TERMINI
- <u>NON</u> RAZIONALIZZARE

- $\lim_{x \to +\infty} \left(\underbrace{\sqrt{2x^2 - 3x}}_{\sqrt{+\infty - \infty}} - \underbrace{\sqrt{5x^2 + 1}}_{\sqrt{+\infty}} \right) =$

$-$ F.I.

RAZIONALIZZAZIONE INVERSA

$(A - B)(A + B) = A^2 - B^2$

$$= \lim_{x \to +\infty} \left(\sqrt{2x^2 - 3x} - \sqrt{5x^2 + 1} \right) \frac{\sqrt{2x^2 - 3x} + \sqrt{5x^2 + 1}}{\sqrt{2x^2 - 3x} + \sqrt{5x^2 + 1}} =$$

$+\infty \quad + \quad \infty$

DIFF. DI QUADRATI

$$= \lim_{x \to +\infty} \frac{2x^2 - 3x - (5x^2 + 1)}{\sqrt{2x^2 - 3x} + \sqrt{5x^2 + 1}} =$$

$$= \lim_{x \to +\infty} \frac{-3x^2 - 3x - 1}{\sqrt{2x^2 - 3x} + \sqrt{5x^2 + 1}} =$$

RACCOGLIMENTO FORZATO

$$= \lim_{x \to +\infty} \frac{x^2 \left[-3 - \overset{\to 0}{\frac{3}{x}} - \overset{\to 0}{\frac{1}{x^2}} \right]}{\sqrt{x^2 (2 - \underset{\to 0}{\frac{3}{x}})} + \sqrt{x^2 (5 + \underset{\to 0}{\frac{1}{x}})}}$$

-9

$$\sqrt{x^2} = |x|$$

si può evitare di usare $|x|$ perché $x \to +\infty$

$$\simeq \lim_{x \to +\infty} \frac{-3x^2}{\underbrace{x\sqrt{2-\cancel{\frac{3}{x}}} + x\sqrt{5}}_{x\left(\sqrt{2}+\sqrt{5}\right)}}$$

CONFRONTANDO I GRADI DEL NUM. E DEN.

$$\simeq \lim_{x \to +\infty} -\frac{3x^{\cancel{2}}}{\cancel{x}\left(\sqrt{2}+\sqrt{5}\right)} = -3 \cdot (+\infty) = -\infty$$

★ RAPPORTO TRA POLINOMI

$\left(\frac{\infty}{\infty}\right)$

① $$\lim_{x\to+\infty} \frac{3x^3 - 4x^2 + 2}{2x^5 - 3} = \frac{+\infty}{+\infty}$$ $\frac{\text{P. grado 3}}{\text{P. grado 5}}$

$$= \lim_{x\to+\infty} \frac{\cancel{x^3}\left(3 - \frac{4}{x}^{\to 0} + \frac{2}{x^3}^{\to 0}\right)}{x^{\cancel{5}\,2}\left(2 - 3/x^5{}_{\to 0}\right)} \simeq \lim_{x\to+\infty} \frac{1}{x^2} = 0$$

② $$\lim_{x\to+\infty} \frac{3x^4 - 2x^2}{5x + 1} = \frac{\text{P grado 4}}{\text{P grado 1}}$$

$$= \lim_{x\to+\infty} \frac{x^{\cancel{4}\,3}\left(3 - 2/x^2\right)}{\cancel{x}\left(5 + 1/x\right)} = +\infty$$

③ $$\lim_{x\to+\infty} \frac{2x^2 - 3}{3x^2 + 1} = \frac{\text{P grado 2}}{\text{P grado 2}}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}\left(2 - 3/x^2\right)}{\cancel{x^2}\left(3 + 1/x^2\right)} = \frac{2}{3}$$

RAPPORTO TRA I COEFF. DI GRADO MASSIMO

★ COMPORTAMENTO ASINTOTICO

$(x \to \infty)$

es1

$$= \lim_{x \to +\infty} \frac{3x^3 - 4x^2 + 2}{2x^5 - 3} =$$

prevalgono

SI COMPORTA COME

$$\sim \lim_{x \to +\infty} \frac{3\cancel{x^3}}{2x^{\cancel{5}\,2}} = 0$$

$$\left[\frac{1}{\infty} = 0 \right]$$

[SI CONSIDERA IL TERMINE DI GRADO MASSIMO PER CIASCUN POLINOMIO]

PER CUI

$\frac{N(x)}{D(x)}$ prevale il grado del Den

$$\Downarrow \lim_{x \to +\infty} = 0$$

es 2

$$\lim_{x \to +\infty} \frac{3x^4 - 2x^2}{5x+1} \simeq \lim_{x \to +\infty} \frac{3x^{\cancel{4}\,3}}{5\cancel{x}} = +\infty$$

es 3

$$\lim_{x \to +\infty} \frac{2x^2 - 3}{3x^2 + 1} \simeq \lim_{x \to +\infty} \frac{2\cancel{x^2}}{3\cancel{x^2}} = \frac{2}{3}$$

QUINDI

$\frac{N(x)}{D(x)}$ se prevale, come grado il numeratore

$$\Rightarrow \lim_{x \to +\infty} \to \pm\infty$$

$\frac{N(x)}{D(x)}$ se i 2 polinomi sono di pari grado

$$\Rightarrow \lim_{x \to +\infty} \frac{N(x)}{D(x)} = \text{Rapporto fra i coefficienti di grado massimo}$$

$$\bullet \lim_{x \to +\infty} \frac{3x^5 - 4x^2 + 1}{7x^4 - 4} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^5}\left(3 - \frac{4}{x^3} + \frac{1}{x^5}\right)}{\cancel{x^4}\left(7 - 4/x^4\right)} = +\infty$$

R. F. $\quad \frac{4}{x^3} \to 0, \quad \frac{1}{x^5} \to 0, \quad 4/x^4 \to 0$

CIÒ CHE È "PIÙ GRANDE"

$$\simeq \lim_{x \to +\infty} \frac{3\cancel{x^5}}{7\cancel{x^4}} = +\infty$$

C. A.

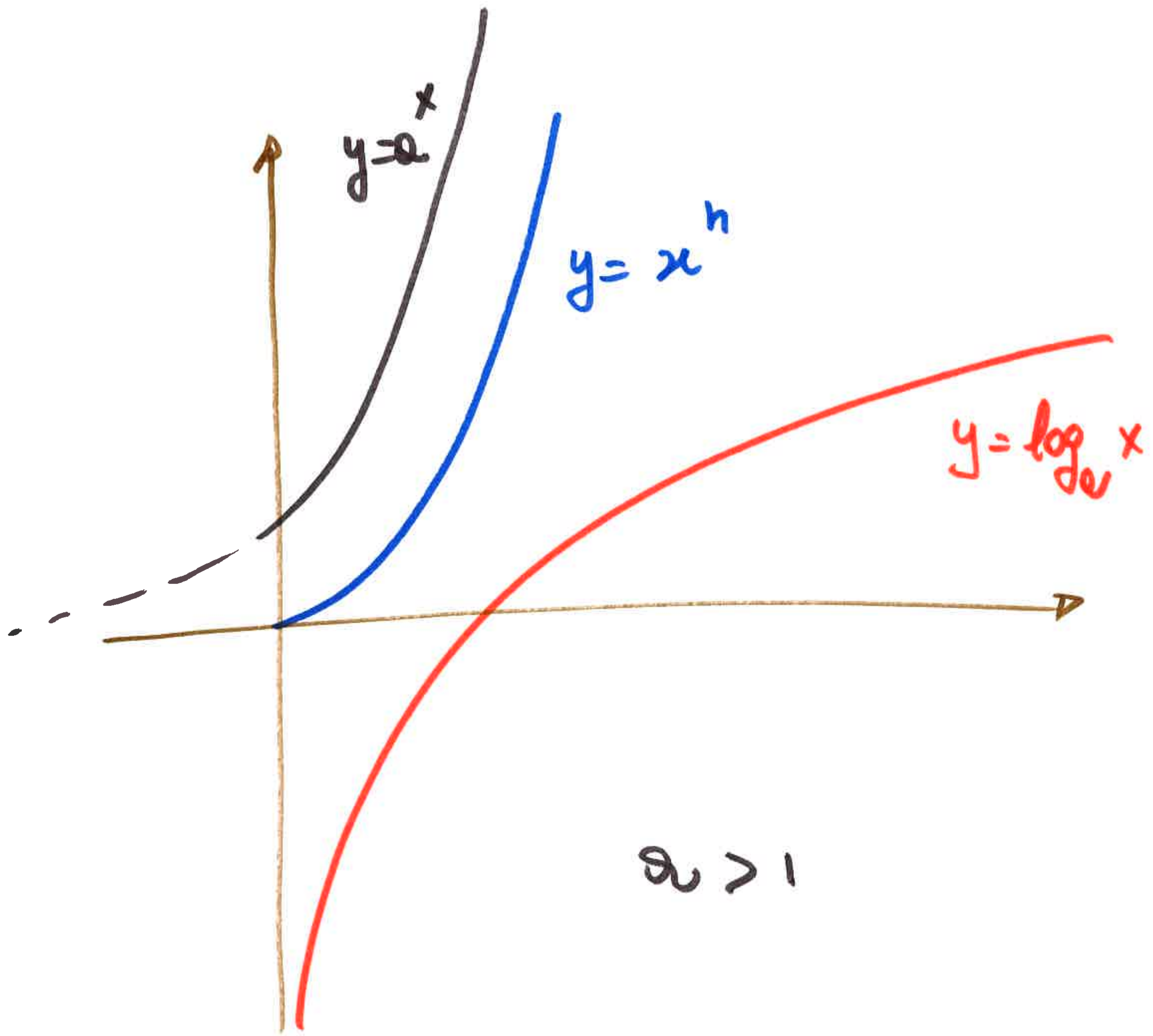
[ciò che è "PIÙ GRANDE"]

$$\lim_{x\to+\infty} \frac{3x^5-4x^2}{7x^5+2x^3}$$

$$\underset{\text{RF.}}{=} \lim_{x\to+\infty} \frac{\cancel{x^5}\left(3-\overbrace{4/x^3}^{\to 0}\right)}{\cancel{x^5}\left(7+\underbrace{\frac{2}{x^2}}_{\to 0}\right)} = \frac{3}{7}$$

$$\simeq \lim_{x\to+\infty} \frac{3\cancel{x^5}}{7\cancel{x^5}} = \frac{3}{7}$$

CATENA DI INFINITI



$$\log_a x < x^n < a^x$$

$$\lim_{x \to +\infty} \frac{\log 5x}{(5x)^2} = 0$$

prevale $(5x)^2$ den

prevale e^{5x} num

$$\lim_{x \to +\infty} \frac{e^{5x}}{\log 5x} = +\infty$$

$$\lim_{x \to +\infty} \frac{x}{e^x} = 0$$

prevale e^x (den)

$$\bullet \lim_{x\to+\infty} \Big(\underbrace{\log_4 \underbrace{(1+2\cdot 4^x)}_{+\infty}}_{+\infty} - \underbrace{x}_{-\infty} \Big) = \text{F.I.}$$

unico $\log_4$ $\quad x = x\cdot\log_4 4 =$

$= \log_4 4^x$

$$= \lim_{x\to+\infty} \left(\log_4 (1+2\cdot 4^x) - \log_4 4^x \right) =$$

DIFF. DI LOG CON LA STESSA BASE

$$= \lim_{x\to+\infty} \left(\log_4 \frac{1+2\cdot 4^x}{4^x} \right) =$$

prevale il termine 4^x

$$\simeq \lim_{x\to+\infty} \log_4 \frac{2\cdot \cancel{4^x}}{\cancel{4^x}} =$$

$$= \log_4 2 = \log_4 \sqrt{4} = \log_4 4^{\frac{1}{2}} = \frac{1}{2}\cdot\underbrace{\log_4 4}_{1} =$$

$$= \frac{1}{2}$$

$$\bullet \lim_{x \to 0^+} \frac{\sqrt{x+\sqrt{x}}}{x^{\frac{1}{4}}} = \frac{0}{0}$$

dom $x>0$

$$\left[\text{N.B.} \quad \lim_{x \to x_0} \frac{P(x)}{Q(x)} = \frac{0}{0} \right.$$

se $P(x_0) = Q(x_0) = 0$

scomporre i polinomi $\Big]$

$$= \lim_{x \to 0^+} \frac{\sqrt{\sqrt{x}\,(\sqrt{x}+1)}}{\sqrt[4]{x}} =$$

$$= \lim_{x \to 0^+} \frac{\cancel{\sqrt[4]{x}}\;\sqrt{\underbrace{\sqrt{x}+1}_{0+1}}}{\cancel{\sqrt[4]{x}}} = 1$$

$$\sqrt{\sqrt{x}} = \sqrt[2\cdot 2]{x} =$$

$$= \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} = x^{\frac{1}{4}}$$

$$\lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} = \frac{1-1}{0} = \frac{0}{0}$$

TOGLIERE LA DIFFERENZA DI RADICI, PERCHÈ DANNO $\emptyset$

$$= \lim_{x\to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \cdot \frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} =$$

$$= \lim_{x\to 0} \frac{1+x-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)} =$$

$$= \lim_{x\to 0} \frac{\cancel{1}+x-\cancel{1}+x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)} =$$

$$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}\left(\sqrt{1+\underset{0}{\underset{\shortparallel}{x}}}+\sqrt{1-\underset{0}{\underset{\shortparallel}{x}}}\right)} = \frac{2}{2} = 1$$

$$\sqrt{1}+\sqrt{1} = 2$$

$$\lim_{x \to \frac{\pi}{6}} \frac{1 + 2\cos\left(\frac{\pi}{2} + x\right)}{1 - 4\sin^2(\pi + x)} =$$

$x = \pi/6$

$$\cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) = \cos\left(\frac{4}{6}\pi\right) = \cos\left(\frac{2}{3}\pi\right) =$$

$$= -\frac{1}{2}$$

$$\operatorname{sen}\left(\pi + \frac{\pi}{6}\right) = \operatorname{sen}\left(\frac{7}{6}\pi\right) = -\frac{1}{2}$$

sostituendo

$$= \frac{1 + 2\cdot\left(-\frac{1}{2}\right)}{1 - 4\cdot\left(-\frac{1}{2}\right)^2} = \frac{1 - 1}{1 - 1} = \frac{0}{0}$$

PROMEMORIA

$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\operatorname{sen}(\pi + x) = -\operatorname{sen} x$$

$$= \lim_{x \to \pi/6} \frac{1 - 2\operatorname{sen} x}{1 - 4\operatorname{sen}^2 x} =$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\cancel{1 - 2\,\text{sen}\,x}}{\cancel{(1-2\,\text{sen}\,x)}\,(1+2\,\text{sen}\,x)} \cdot$$

SCOMPOSIZIONE

$$= \lim_{x \to \frac{\pi}{6}} \frac{1}{1+2\,\text{sen}\,x} =$$

$$= \frac{1}{1+\cancel{2}\,\frac{1}{\cancel{2}}} = \frac{1}{2}$$

- $$\lim_{x \to 0} \left(\sin^2 \frac{1}{x} + \cos^2 \frac{1}{x} \right) = 1$$

$\forall x \in \mathbb{R}$

[Relazione fond.]

ATTENZIONE :

$$\lim_{x \to 0} \text{sen}^2 \frac{1}{x} = \nexists$$

$$\lim_{x \to 0} \cos^2 \frac{1}{x} = \nexists$$

$$\lim_{x \to 1} \frac{\sqrt{x}-1}{x^2-1} = \frac{1-1}{1-1} = \frac{0}{0}$$

SCOMPORRE

$$= \lim_{x \to 1} \frac{\sqrt{x}-1}{(x-1)(x+1)} = \frac{1-1}{(1-1)(1+1)}$$

$x-1 =$ DIFF. DI QUADRATI

$$x = (\sqrt{x})^2$$

$$= \lim_{x \to 1} \frac{\cancel{\sqrt{x}-1}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)(x+1)} =$$

$$= \lim_{x \to 1} \frac{1}{(\sqrt{x}+1)(x+1)} = \frac{1}{(1+1)(1+1)} =$$

$$= \frac{1}{4}$$

$$\lim_{x \to 1} \frac{3x^3 - 3}{(x-1)^2} =$$

$$= \frac{3-3}{(1-1)^2} = \frac{0}{0} \quad \text{F. I.}$$

se $x \to 1$ è zero per i polinomi N, D.

$(x-1)$ è divisore $\Rightarrow$

✓ scomposizione

✓ divisione

$$= \lim_{x \to 1} \frac{3(x^3-1)}{(x-1)^2} =$$

differenza di cubi

$$= \lim_{x \to 1} \frac{3\cancel{(x-1)}(x^2+1+x)}{(x-1)^{\cancel{2}}} =$$

SOSTITUISCO

$$= 3\,\frac{1+1+1}{1-1} = \frac{9}{0} = \infty$$

MEGLIO SPECIFICARE

$$\lim_{x \to 1} \frac{3\overbrace{(x^2+1+x)}^{+}}{x-1}$$

segno è dato de$(x-1)$

segno $(x-1) \geq 0$

$$\lim_{x \to 1^{+}} \frac{3(x^2+1+x)}{\underbrace{x-1}_{+}} = \frac{9}{0^{+}} = +\infty$$

DESTRA
Den > 0

$$\lim_{x \to 1^{-}} \frac{3(x^2+1+x)}{\underbrace{x-1}_{-}} = \frac{9}{0^{-}} = -\infty$$

SINISTRA
Den < 0

$$\lim_{x \to +\infty} \frac{\sqrt{1+x} - \sqrt{x}}{x} =$$

$$\text{DOM} \begin{cases} 1+x \geq 0 \\ x \geq 0 \\ x \neq 0 \end{cases} \Rightarrow \underline{\underline{x > 0}}$$

$$= \frac{\sqrt{1+\infty} - \sqrt{+\infty}}{+\infty} = \frac{+\infty - \infty^{*}}{+\infty}$$

TRASFORMARE IN UNA SOMMA DI RADICI $(*)$

$$(A-B) \cdot (A+B) = A^2 - B^2$$

$$= \lim_{x \to +\infty} \frac{\sqrt{1+x} - \sqrt{x}}{x} \cdot \frac{\sqrt{1+x} + \sqrt{x}}{\sqrt{1+x} + \sqrt{x}} =$$

$$= \lim_{x \to +\infty} \frac{(\cancel{\sqrt{1+x}})^{\cancel{2}} - (\cancel{\sqrt{x}})^{\cancel{2}}}{x \cdot (\sqrt{1+x} + \sqrt{x})} =$$

$$= \lim_{x \to +\infty} \frac{1 + x - x}{x(\sqrt{1+x} + \sqrt{x})} = \frac{1}{+\infty} = 0$$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{x}}{x} =$$

$$= \frac{\sqrt{1+0} - \sqrt{0}}{0} = \frac{1}{0} = \infty \quad \pm\infty$$

NON CORRETTO!

DOMINIO $x > 0$

$$\lim_{x \to 0^+} \frac{\sqrt{1+x} - \sqrt{x}}{x}$$

$$\lim_{x \to -\infty} \frac{\sqrt{1+x} - \sqrt{x}}{x}$$

NON ESISTE!

DOMINIO

$$\begin{cases} x \neq 0 \\ x+1 \geq 0 \\ x \geq 0 \end{cases} \qquad \boxed{x > 0}$$

$$\lim_{x \to 0^+} \frac{\sqrt{1+x} - \sqrt{x}}{x} = +\infty$$

$\bullet \lim_{x\to+\infty} (\underline{2}^x - x^2) = +\infty - \infty$

RACCOLGO 2^x (più veloce)

$$= \lim_{x\to+\infty} 2^x \left(1 - \underbrace{\frac{x^2}{2^x}}_{\to 0}\right) = +\infty$$

$$\bullet \lim_{x\to+\infty} \frac{2^x - x^2}{x^2 + \log^2 x} = \frac{+\infty - \infty}{+\infty + \infty}$$

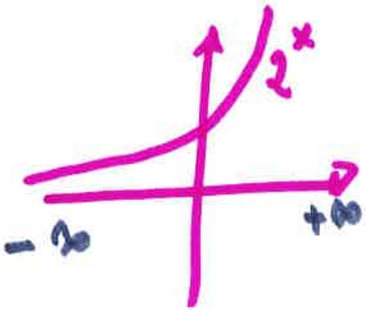
$$= \lim_{x\to+\infty} \frac{2^x \left(1 - \frac{x^2}{2^x}\right)^{\to 0}}{x^2 \left(1 + \frac{\log^2 x}{x^2}\right)} =$$

$$\frac{\log^2 x}{x^2} = \left(\frac{\log x}{x}\right)^2 \to 0$$

prevale 2^x

$= +\infty$

• $\lim\limits_{x\to-\infty} \left(2^x - x^2\right) =$



$= 2^{-\infty} - (-\infty)^2 =$

$= 0 - \infty = -\infty$

• $\lim\limits_{x\to+\infty} \underbrace{\left(x - \ln\left(1+4e^x\right)\right)}_{+\infty - \ln(+\infty)} =$

$= \lim\limits_{x\to+\infty} \left(x - \ln\left[4e^x\left(\underbrace{\frac{1}{4e^x}}_{\to 0} + 1\right)\right]\right) =$

$\simeq \lim\limits_{x\to+\infty} \left(x - \ln\left(4e^x\right)\right) =$

$= \lim\limits_{x\to+\infty} \left(x - \ln 4 - \ln e^x\right) =$

$$= \lim_{x\to+\infty} \left(x - \ln 4 - x \cdot \underbrace{\ln e}_{1} \right) =$$

$$= -\ln 4$$

• $$\lim_{x\to 0^+} \frac{\log(x+x^2)}{\log x} = \frac{\log 0^+}{\log 0^+} = \frac{-\infty}{-\infty}$$

$$= \lim_{x\to 0^+} \frac{\log(x\cdot(1+x))}{\log x} =$$

$$= \lim_{x\to 0^+} \frac{\log x + \log(1+x)}{\log x} =$$

$$= \lim_{x\to 0^+} \left[1 + \frac{\log(1+x)}{\log x} \right] =$$

$$= 1 + \lim_{x\to 0^+} \left[\underbrace{\frac{\log(1+x)}{x}}_{\to 1} \cdot \overbrace{\frac{x}{\log x}}^{\to 0} \right] = 1$$

→ LIMITE NOTEVOLE →

• Ricordare

$$\lim_{x \to 0^+} \underset{\sim}{x}^{a} (\lg_b x)^{c} = 0$$

$$\forall \; a, b > 0 \quad b \neq 1$$
$$c \in \mathbb{R}$$

$$\Rightarrow y = (\log_b x)^{c}$$

equivale, ad esempio, a

$$y = (\log_2 x)^{3}$$

$$\lim_{x \to 0^+} y \Rightarrow -\infty$$

$$y = (\log_{\frac{1}{2}} x)^{3}$$

$$\lim_{x \to 0^+} y = +\infty$$

$$\lim_{x \to 0^+} x^2 \ln x = 0$$

prevale

(Ricordarsi gli ordini di infinito

· Passaggio all'esponenziale

$$\lim_{x \to 0} (g(x))^{f(x)} = \text{F.I}$$

$$= \lim_{x \to 0} e^{\log (g(x))^{f(x)}} =$$

$$= \lim_{x \to 0} e^{\overbrace{f(x) \cdot \log(g(x))}^{\text{F.I.}}}$$

base costante

es.

$$\lim_{x \to 0^+} x^x = \text{F.I.}$$

$$= \lim_{x \to 0^+} e^{\log x^x} =$$

$$= \lim_{x \to 0^+} e^{\overbrace{x \cdot \log x}^{\to 0}} = e^0 = 1$$

$$\lim_{x \to +\infty} \left(f(x) \right)^{g(x)} = F.I$$

$$= \lim_{x \to +\infty} e^{\log(f(x))^{g(x)}} =$$

$$= \lim_{x \to +\infty} e^{\underbrace{g(x) \cdot \log(f(x))}_{\text{esponente}}}$$

es.

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = F.I.$$

$$= \lim_{x \to +\infty} e^{\log x^{\frac{1}{x}}} =$$

$$= \lim_{x \to +\infty} e^{\frac{\log x}{x}} = e^0 = 1$$

F.I

prevale x

$$\cdot \lim_{x \to +\infty} \frac{2 + \sin x}{\ln x} = 0$$

N. $2 + [-1, 1] \to [1, 3]$

D $\ln x \to +\infty$

$$-1 \leq \sin x \leq 1$$

$$\frac{1 = 2 - 1}{\ln x} \leq \frac{2 + \sin x}{\ln x} \leq \frac{2 + 1 = 3}{\ln x}$$

$x \to +\infty$ $\quad \frac{1}{\ln x} \to 0$ $\qquad \frac{3}{\ln x} \to 0$

$$\cdot \lim_{x \to +\infty} \frac{x \cos x}{2^x - x^2} =$$

$$= \lim_{x \to +\infty} \frac{\boxed{x} \cdot \overset{[-1, 1]}{\cos x}}{\boxed{2^x} \left(1 - \underbrace{\frac{x^2}{2^x}}_{\to 0}\right)} = 0$$

prevale 2^x

· INFINITESIMI (LOCALI)

1) Se $f(x) \le g(x)$ in $I(x_0)$

$$\lim_{x \to x_0} f(x) \le \lim_{x \to x_0} g(x)$$

[estendo i limiti

2) $f(x)$ limitata in $I(x_0)$

$g(x)$ infinitesima per $x \to x_0$

$\Rightarrow f(x) \cdot g(x)$ è infinitesima per $x \to x_0$

$$|f(x)| \le M \quad \forall x \in I(x_0)$$

$$\lim_{x \to x_0} g(x) = 0$$

$$\Rightarrow \lim_{x \to x_0} f(x) \cdot g(x) = 0$$

Limiti notevoli

$$\lim_{x \to +\infty} \left(1+\frac{1}{x}\right)^x = e$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\cdot \lim_{x \to 0} \frac{\sin x}{x} = 1 \to \lim_{n \to \infty} \frac{\sin \frac{1}{n}}{\frac{1}{n}} = 1$$

$$\lim_{f(x) \to 0} \frac{\sin(f(x))}{f(x)} = 1$$

$$\lim_{x \to -\infty} \frac{\sin^2 e^{x \to 0}}{e^{x} \to 0} = \frac{\sin^2 0}{0}$$

$$= \lim_{x \to -\infty} \underbrace{\frac{\sin e^x}{e^x}}_{1} \cdot \sin e^x = 0$$

$\sin(0) = 0$

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \underbrace{\frac{\sin 2x}{2x}}_{1} \cdot 2\cancel{x} \; \frac{1}{\underbrace{\frac{\sin 3x}{3x}}_{1} \cdot 3\cancel{x}} = \frac{2}{3}$$

$$\lim_{x \to 0} \frac{\sin ax}{x^3} = \frac{0}{0} \qquad a \neq 0$$

$\forall a \in \mathbb{R}_0$

$$= \lim_{x \to 0} \underbrace{\frac{\sin ax}{ax}}_{1} \cdot \frac{a\cancel{x}}{x^{\cancel{3}2}} =$$

$$= \lim_{x \to 0} \frac{a}{x^2} \to 0^+ \begin{cases} +\infty & a > 0 \\ -\infty & a < 0 \end{cases}$$

$$\lim_{x \to \pi} \frac{1+\cos x}{(\pi - x)^2} =$$

$$= \frac{1+\cos\pi}{(\pi-\pi)^2} = \frac{1-1}{0} = \frac{0}{0}$$

$\pi - x = t$

$\pi \quad 0$

$x = \pi - t$

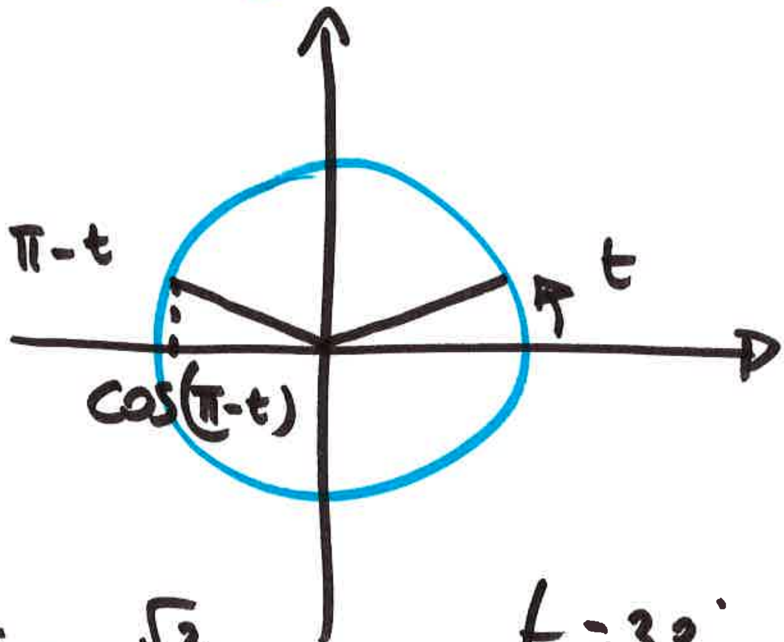
$$= \lim_{t \to 0} \frac{1+\cos(\pi - t)}{t^2} =$$

$$= \lim_{t \to 0} \frac{(1-\cos t)}{t^2} \cdot \frac{1+\cos t}{1+\cos t} =$$

$$\lim_{x \to +\infty} \frac{\sin^2 e^x}{e^x} =$$

$$= \frac{\sin^2 (e^{+\infty})}{e^{+\infty}} = \frac{\sin^2(+\infty)}{+\infty}$$

$$= \frac{[0, 1]}{+\infty} = 0$$



$\cos \frac{5}{6}\pi = -\frac{\sqrt{3}}{2}$

$t = 30°, \ \frac{\pi}{6}$

$\pi - \frac{\pi}{6} = \frac{5}{6}\pi$

$-$ sen

$$= \lim_{t \to 0} \frac{1-\cos^2 t}{t^2(1+\cos t)} =$$

$$= \lim_{t \to 0} \frac{\operatorname{sen}^2 t}{t^2\,(1+\cos t)} = \frac{1}{2}$$

$1+1$

1

Derivati

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

$$\lim_{f(x) \to 0} \frac{1-\cos f(x)}{(f(x))^2} = \frac{1}{2}$$

$$\lim_{f(x) \to 0} \frac{\operatorname{Tg} f(x)}{f(x)} = 1$$

$$\lim_{x \to 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\arctan x}{x} = 1$$

$$\lim_{f(x) \to 0} \frac{\arcsin f(x)}{f(x)} = 1$$

$$\lim_{f(x) \to 0} \frac{\arctan f(x)}{f(x)} = 1$$

$$\lim_{x \to 0} \frac{x^2 + x \operatorname{sen} x}{1 - \cos x} = \frac{0}{0}$$

(Racc. fattore)

$$= \lim_{x \to 0} \frac{x^2 \left(1 + \frac{\operatorname{sen} x}{x}\right)}{1 - \cos x} =$$

$$= \lim_{x \to 0} \frac{1 + \underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1}}{\underbrace{\frac{1 - \cos x}{x^2}}_{\to 1/2}} =$$

$$= \frac{2}{1/2} = 4$$

$$\lim_{x \to 0} \frac{\operatorname{tg} x - \operatorname{sen} x}{x^3} = \frac{0}{0}$$

$$N = \frac{\operatorname{sen} x - \operatorname{sen} x \cdot \cos x}{\cos x}$$

$$= \lim_{x \to 0} \frac{\frac{\operatorname{sen} x}{\cos x} - \operatorname{sen} x}{x^3} =$$

$$= \lim_{x \to 0} \frac{\operatorname{sen} x \, (1 - \cos x)}{\cos x \cdot x^3} =$$

$$= \lim_{x \to 0} \underset{\substack{\downarrow \\ 1}}{\frac{\operatorname{sen} x}{x}} \cdot \underbrace{\frac{1 - \cos x}{x^2}}_{1/2} \cdot \underset{\substack{\downarrow \\ 1}}{\frac{1}{\cos x}} = \frac{1}{2}$$