E A

Lezione parte 2

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- Ricordare

$$\lim_{x\to 0^+} x^a (\lg_b x)^c = 0$$

$\forall\ a, b > 0 \quad b \neq 1$

$c \in \mathbb{R}$

$\Rightarrow y = (\log_b x)^c$

equivale, ad esempio, a

$$y = (\log_2 x)^3$$

$$\lim_{x\to 0^+} y \Rightarrow -\infty$$

$$y = (\log_{\frac{1}{2}} x)^3$$

$$\lim_{x\to 0^+} y = +\infty$$

- $$\lim_{x \to 0^+} \underset{\substack{\uparrow \\ \text{prevale}}}{x^2} \ln x = 0$$

(Ricordarsi gli ordini di infinito

. Passaggio all'esponenziale

$$\lim_{x\to 0} (g(x))^{f(x)} = F.I$$

$$= \lim_{x\to 0} e^{\log (g(x))^{f(x)}} =$$

$$= \lim_{x\to 0} e^{\overbrace{f(x) \cdot \log (g(x))}^{F.I.}}$$

base costante

es.

$$\lim_{x\to 0^+} x^{x} = F.I.$$

$$= \lim_{x\to 0^+} e^{\log x^{x}} =$$

$$= \lim_{x\to 0^+} e^{\overbrace{x \cdot \log x}^{\to 0}} = e^{0} = 1$$

$$\underset{x\to+\infty}{\ell}\left(f(x)\right)^{g(x)} = \text{F.I}$$

$$= \underset{x\to+\infty}{\ell} e^{\log(f(x))^{g(x)}} =$$

$$= \underset{x\to+\infty}{\ell} e^{\underbrace{g(x)\cdot\log(f(x))}_{\text{esponente}}}$$

es.

$$\underset{x\to+\infty}{\ell} x^{\frac{1}{x}} = \text{F.I.}$$

$$= \underset{x\to+\infty}{\ell} e^{\log x^{\frac{1}{x}}} =$$

$$= \underset{x\to+\infty}{\ell} e^{\frac{\log x}{x}} \quad \text{F.I} \quad = e^{0} = 1$$

prevale x

- $$\lim_{x\to+\infty} \frac{2+\operatorname{sen} x}{\ln x} = 0$$

N. $2+[-1,1] \to [1,3]$

D $\ln x \to +\infty$

$$-1 \le \operatorname{sen} x \le 1$$

$$x\to+\infty \quad \underset{\to 0}{\frac{1=2-1}{\ln x}} \le \frac{2+\operatorname{sen} x}{\ln x^{+}} \le \underset{\downarrow 0}{\frac{2+1=3}{\ln x}}$$

- $$\lim_{x\to+\infty} \frac{x\cos x}{2^x - x^2} =$$

$$= \lim_{x\to+\infty} \frac{\boxed{x}\;\overset{[-1,1]}{\cos x}}{\boxed{2^x}\left(1-\underset{\downarrow 0}{\frac{x^2}{2^x}}\right)} = 0$$

prevale 2^x

. INFINITESIMI (LOCALI)

1) Se $f(x) \leq g(x)$ in $I(x_0)$

$$\lim_{x \to x_0} f(x) \leq \lim_{x \to x_0} g(x)$$

[esistendo i limiti

2) $f(x)$ limitata in $I(x_0)$

$g(x)$ infinitesima per $x \to x_0$

$\Rightarrow f(x) \cdot g(x)$ è infinitesima per $x \to x_0$

$|f(x)| \leq M \quad \forall x \in I(x_0)$

$$\lim_{x \to x_0} g(x) = 0$$

$$\Rightarrow \lim_{x \to x_0} f(x) \cdot g(x) = 0$$

Limiti notevoli

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1$$

$$\cdot \lim_{x \to 0} \frac{\operatorname{sen} x}{x} = 1 \to \lim_{n \to \infty} \frac{\operatorname{sen} \frac{1}{n}}{\frac{1}{n}} = 1$$

$$\lim_{f(x) \to 0} \frac{\sin(f(x))}{f(x)} = 1$$

$$\lim_{x \to -\infty} \frac{\sin^2 e^{x} \nearrow 0}{e^{x} \searrow 0} = \frac{\operatorname{sen}^2 0}{0}$$

$$= \lim_{x \to -\infty} \underbrace{\frac{\operatorname{sen} e^{x}}{e^{x}}}_{1} \cdot \underset{\operatorname{sen}(0) = 0}{\operatorname{sen} e^{x}} = 0$$

$$\lim_{x \to 0} \frac{\sin 2x}{\sin 3x} = \frac{0}{0}$$

$$= \lim_{x \to 0} \underbrace{\frac{\sin 2x}{2x}}_{1} \cdot 2\cancel{x} \, \frac{1}{\underbrace{\frac{\sin 3x}{3x}}_{1} \cdot 3\cancel{x}} = \frac{2}{3}$$

$$\lim_{x\to 0} \frac{\sin ax}{x^3} = \frac{0}{0}$$

$a \neq 0$

$\forall a \in \mathbb{R}_0$

$$= \lim_{x\to 0} \underbrace{\frac{\sin ax}{ax}}_{1} \cdot \frac{a\cancel{x}}{x^{\cancel{3}2}} =$$

$$= \lim_{x\to 0} \frac{a}{x^2} \quad x^2 \to 0^+ \quad \begin{cases} +\infty & a > 0 \\ -\infty & a < 0 \end{cases}$$

$$\lim_{x\to \pi} \frac{1+\cos x}{(\pi - x)^2} =$$

$$= \frac{1+\cos\pi}{(\pi-\pi)^2} = \frac{1-1}{0} = \frac{0}{0}$$

$$\begin{array}{ccc} \pi - x & = & t \\ \downarrow & & \downarrow \\ \pi & & 0 \end{array}$$

$x = \pi - t$

$$= \lim_{t\to 0} \frac{1+\cos(\pi - t)}{t^2} =$$

$$= \lim_{t\to 0} \frac{(1-\cos t)}{t^2} \cdot \frac{1+\cos t}{1+\cos t} =$$

$$\lim_{x \to +\infty} \frac{\sin^2 e^x}{e^x} =$$

$$= \frac{\sin^2 (e^{+\infty})}{e^{+\infty}} = \frac{\sin^2(+\infty)}{+\infty}$$

$$= \frac{[0, 1]}{+\infty} = 0$$

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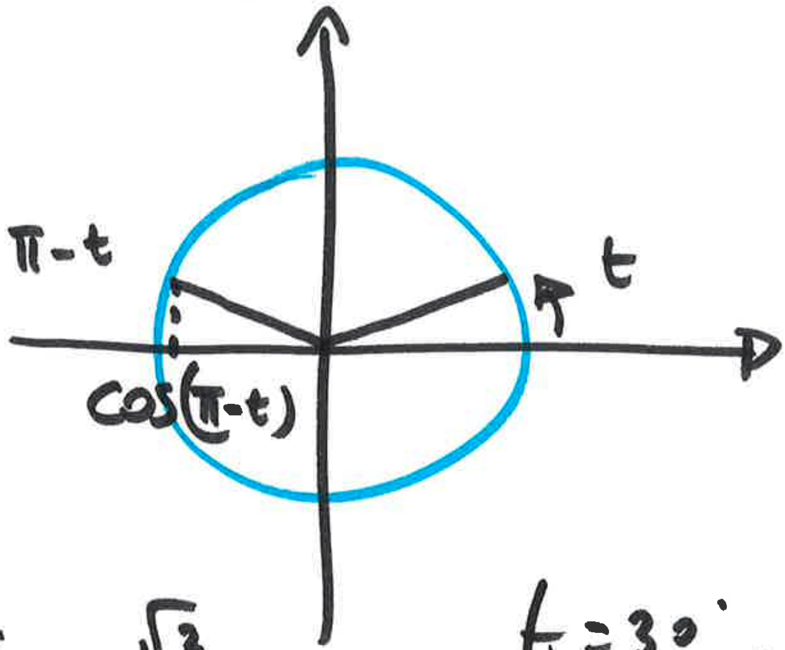


$\cos \frac{5}{6}\pi = -\frac{\sqrt{3}}{2}$

$t = 30°, \ \frac{\pi}{6}$

$\pi - \frac{\pi}{6} = 5/6\ \pi$

- sen

$$= \underset{t\to 0}{\ell} \; \frac{1-\cos^2 t}{t^2(1+\cos t)} =$$

$$= \underset{t\to 0}{\ell} \; \frac{\operatorname{sen}^2 t}{t^2 \underbrace{(1+\cos t)}_{1+1}} = \frac{1}{2}$$

$$\downarrow 1$$

Derivati

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\operatorname{Tan} x}{x} = 1$$

$$\lim_{f(x) \to 0} \frac{1-\cos f(x)}{\left(f(x)\right)^2} = \frac{1}{2}$$

$$\lim_{f(x) \to 0} \frac{\operatorname{Tg} f(x)}{f(x)} = 1$$

$$\lim_{x \to 0} \frac{\arcsin x}{x} = 1$$

$$\lim_{x \to 0} \frac{\arctan x}{x} = 1$$

$$\lim_{f(x) \to 0} \frac{\arcsin f(x)}{f(x)} = 1$$

$$\lim_{f(x) \to 0} \frac{\arctan f(x)}{f(x)} = 1$$

$$\bullet \lim_{x \to 0} \frac{x^2 + x \operatorname{sen} x}{1 - \cos x} = \frac{0}{0}$$

(Racc. forzato)

$$= \lim_{x \to 0} \frac{x^2 \left(1 + \frac{\operatorname{sen} x}{x}\right)}{1 - \cos x} =$$

$$= \lim_{x \to 0} \frac{\left. 1 + \frac{\operatorname{sen} x}{x} \right\} \rightarrow 1}{\left. \frac{1 - \cos x}{x^2} \right\} \rightarrow \frac{1}{2}} =$$

$$= \frac{2}{1/2} = 4$$

$$\lim_{x \to 0} \frac{tg\,x - sen\,x}{x^3} = \frac{0}{0}$$

$$N = \frac{\overbrace{sen\,x} - \overbrace{sen\,x \cdot cos\,x}}{cos\,x}$$

$$= \lim_{x \to 0} \frac{\frac{sen\,x}{cos\,x} - sen\,x}{x^3} =$$

$$= \lim_{x \to 0} \frac{sen\,x\,(1 - cos\,x)}{cos\,x \cdot x^3} =$$

$$= \lim_{x \to 0} \underset{\downarrow\, 1}{\frac{sen\,x}{x}} \cdot \underbrace{\frac{1 - cos\,x}{x^2}}_{1/2} \cdot \underset{\downarrow\, 1}{\frac{1}{cos\,x}} = \frac{1}{2}$$

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x}=e}$$

$$\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$$

$$\lim_{f(x)\to\pm\infty}\left(1+\frac{1}{f(x)}\right)^{f(x)}=e$$

$$\lim_{f(x)\to 0}\frac{\ln(1+f(x))}{f(x)}=1$$

$$=\frac{\ln(1+x)}{\ln b}$$

$$\lim_{x\to 0}\frac{\log_b(1+x)}{x}=\frac{1}{\ln b}$$

(base Trasformata in ln)

$b>0$ $\neq 1$

- $$\lim_{x\to 0} \frac{e^{x}-1}{x} = 1$$

$$\lim_{f(x)\to 0} \frac{e^{f(x)}-1}{f(x)} = 1$$

$$\lim_{x\to 0} \frac{4^{x}-1}{2x} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{e^{\ln 4^{x}}-1}{2x} =$$

$$= \lim_{x\to 0} \underbrace{\frac{e^{x\ln 4}-1}{2x\cdot \ln 4}}_{\to 1}\cdot \ln 4 = \frac{\ln 4}{2} =$$

$$= \frac{\ln 2^{2}}{2} = 1\cdot \ln 2$$

passaggio esp.

$$\left[\; b>0 \qquad \frac{b^{x}-1}{x} = \frac{e^{x\ln b}-1}{x\ln b}\cdot \ln b \;\right]$$

$$\lim_{x\to 0} \left(1+\operatorname{sen} x\right)^{\frac{1}{x}} = 1^{\infty}$$

$$= \lim_{x\to 0} e^{\ln\left(1+\operatorname{sen} x\right)^{\frac{1}{x}}} =$$

$$= \lim_{x\to 0} e^{\frac{1}{x}\ln\left(1+\operatorname{sen} x\right)} =$$

$$= \lim_{x\to 0} e^{\frac{\ln\left(1+\overset{\to 0}{\operatorname{sen} x}\right)}{x\,\operatorname{sen} x}\cdot\operatorname{sen} x} =$$

$$= \lim_{x\to 0} e^{\underbrace{\frac{\ln\left(1+\operatorname{sen} x\right)}{\operatorname{sen} x}}_{\to 1}\cdot\underbrace{\frac{\operatorname{sen} x}{x}}_{\to 1}} =$$

$$= e^{1} = e$$

$$\cdot \lim_{x \to 0} \frac{e^{x^3} - 1}{x^2 \cdot \operatorname{Tg} x} = \frac{1-1}{0 \cdot 0} = \frac{0}{0}$$

$$= \lim_{x \to 0} \frac{e^{x^3} - 1}{x^2 \cdot x} \cdot \frac{1}{\frac{\operatorname{Tg} x}{x}} =$$

Moltiplicato e diviso per x il denominatore

$$= \lim_{x \to 0} \underbrace{\frac{e^{x^3} - 1}{x^3}}_{\downarrow 1} \cdot \frac{1}{\underbrace{\frac{\operatorname{Tg} x}{x}}_{1}} = 1$$

Altri:

$$\lim_{x \to 0} \frac{(1+x)^a - 1}{x} = a$$

$$\lim_{x \to 0} \frac{\sinh x}{x} = 1$$

$$\lim_{x \to 0} \frac{\cosh x - 1}{x^2} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\tanh x}{x} = 1$$

così per $f(x) \to 0$

$$\lim_{x\to+\infty}\left(1-2^{-x}\log 3\right)^{2^{x+1}} =$$

$$\xleftarrow{\text{Arg log}}$$

$$\log\left(1-2^{-x}\log 3\right)^{2^{x+1}}$$

$$= \lim_{x\to+\infty} e^{2^{x+1}\cdot\log\left(1-2^{-x}\cdot\log 3\right)} =$$

$$= \lim_{x\to+\infty} \boxed{e} \qquad = e^{----}$$

$$(*)\ \lim_{x\to+\infty} 2^{x+1}\cdot\log\left(1-\frac{\log 3}{2^{x}}\right) =$$

$$= \lim_{x\to+\infty} 2^{x}\cdot 2\ \log\left(1-\frac{\log 3}{2^{x}}\right) =$$

$$= \lim_{x\to+\infty} \textcircled{2}\ \log\left(1\ominus\frac{\textcircled{\log 3}}{2^{x}}\right)^{2^{x}} =$$

$$\left(1+\frac{1}{x}\right)^{x} \qquad x \to \infty$$

$$\searrow 2^{x}$$

$$= 2\cdot\lim_{x\to+\infty} \log\left(1+\frac{1}{-\frac{2^{x}}{\log 3}}\right)^{-\frac{2^{x}}{\log 3}\cdot\left(-\log 3\right)} \qquad t = -\frac{2^{x}}{\log 3}$$

$$= 2\cdot\lim_{t\to+\infty} \log\left[\underbrace{\left(1+\frac{1}{t}\right)^{t}}_{e}\right]^{\cdot\left(-\log 3\right)}$$

$$= 2\cdot\log\left(e^{-\log 3}\right) =$$

$$= 2\left(-\log 3\right)\cdot\underbrace{\cancel{\log e}}_{1} =$$

$$= \log\left[3^{-2}\right] = \log\left(1/9\right)$$

ESPONENTE DI e

$$= e^{\lim_{x\to+\infty}\cdots} = e^{\log\frac{1}{9}} = 1/9$$

$$\lim_{x\to+\infty} \frac{e^{\frac{\log^2 x+\log x}{\log x-1}}}{x} =$$

TRASFORMAZIONE ALL'ESPONENZIALE $x = e^{\log x}$

$$= \lim_{x\to+\infty} \frac{e^{\frac{\log^2 x+\log x}{\log x-1}}}{e^{\log x}} =$$

prop. potenze

$$= \lim_{x\to+\infty} e^{\frac{\log^2 x+\log x}{\log x-1} - \log x} =$$

$$= \lim_{x\to+\infty} e^{\frac{\cancel{\log^2 x} + \log x - \cancel{\log^2 x} + \log x}{\log x -1}} =$$

$$= \lim_{x\to+\infty} e^{\frac{2\boxed{\log x}}{\boxed{\log x}-1}} = e^2$$

Rapporto tra i coeff. grado massimo

$$\frac{2 \log x}{\log x - 1} =$$

$$= \frac{2 \cancel{\log x}}{\cancel{\log x}\left(1 - \underset{\to 0}{\frac{1}{\log x}}\right)}$$

Polinomi

$$\lim_{x \to +\infty} \frac{\boxed{x^2} + 1}{\boxed{3x^2} + 1} =$$

$$\simeq \lim_{x \to +\infty} \frac{x^2}{3x^2} = \frac{1}{3}$$

$x \to +\infty$

$$\log x \simeq \log(x+1) =$$

$$= \log\left[x \cdot \left(1+\frac{1}{x}\right)\right] =$$

$$= \log x + \underbrace{\log\left(1+\underbrace{\frac{1}{x}}_{\to 0}\right)}_{\to 0}$$

il comportamento
di $y = \log x$
$y = \log(x+1)$
è asintotico

$$\cdot \lim_{x \to +\infty} \left(\log_3 \left(1 + 3^{x+2} \right) - 3x \right) =$$

Trasformare in un unico log.

$$= \lim_{x \to +\infty} \left(\log_3 \left(1 + 3^{x+2} \right) - 3x \cdot \log_3 3 \right) =$$

$$= \lim_{x \to +\infty} \left(\log_3 \left(1 + 3^{x+2} \right) - \log_3 3^{3x} \right) =$$

DIFF. LOG.

$$= \lim_{x \to +\infty} \left(\log_3 \left(\frac{1 + 3^{x+2}}{3^{3x}} \right) \right) =$$

$$= \log_3 \left\{ \lim_{x \to +\infty} \left(\frac{1}{3^{3x}} + \frac{3^{x+2}}{3^{3x}} \right) \right\} =$$

$$= \log_3 \left\{ \lim_{x \to +\infty} \left(\frac{1}{3^{3x}} + 3^{x+2-3x} \right) \right\} =$$

$$= \log_3 \left\{ \lim_{x \to +\infty} \left(\underbrace{\frac{1}{3^{3x}}}_{\frac{1}{\infty} \rightsquigarrow 0} + \underbrace{3^{-2x+2}}_{3^{-\infty} \rightsquigarrow 0} \right) \right\}$$

$$= \log_3 0 = -\infty$$

$$\bullet \lim_{x\to+\infty}\left(\frac{\sqrt[4]{x^2+3x+1}-\sqrt{x}}{\sqrt{x}+7\ln^2 x}\right)\cdot\left(\frac{3x^2+1}{x-\cos x}\right)^{x^{\frac{7}{x}}}$$

1^a parte

$$\frac{\sqrt[4]{x^2+3x+1}-\sqrt{x}}{\sqrt{x}+7\ln^2 x}\cdot\frac{\sqrt[4]{x^2+3x+1}+\sqrt{x}}{\sqrt[4]{x^2+3x+1}+\sqrt{x}}=$$

$A-B$ $\qquad A+B$

$$A^2=\left(\sqrt[\boxed{4}]{x^2+3x+1}\right)^{\boxed{2}}=\sqrt{x^2+3x+1}$$

prop. radicali

$$B^2=\left(\sqrt{x}\right)^2=x$$

F. I.

$$=\frac{\sqrt{x^2+3x+1}-x}{\left(\sqrt{x}+7\ln^2 x\right)\left(\sqrt[4]{x^2+3x+1}+\sqrt{x}\right)}\cdot\frac{\sqrt{x^2+3x+1}+x}{\sqrt{x^2+3x+1}+x}=$$

grado 1

$$=\frac{\cancel{x^2}+3x+1-\cancel{x^2}}{\left(\sqrt{x}+7\ln^2 x\right)\left(\sqrt[4]{x^2+3x+1}+\sqrt{x}\right)\left(\sqrt{\boxed{x^2}+3x+1}\boxed{+x}\right)}$$

$x^{2/4}=x^{1/2}$ $\qquad x^{\frac{1}{2}}$

grado $1/2$ $\qquad$ 1° grado

$$\sqrt{x} + 7 \ln^2 x \quad =$$

$$= \sqrt{x} \quad \left(1 + \frac{\ln^2 x}{\sqrt{x}} \right)$$

$$\rightsquigarrow 0$$

1e Parte

$$\triangleq \frac{3x}{x^{\frac{1}{2}} \left(2 x^{\frac{1}{2}}\right) \left(2x\right)}$$

2' parte

$$\left(\frac{3x^2+1}{x-\cos x}\right)^{x^{\frac{7}{x}}} =$$

$$\log\left(\frac{3x^2+1}{x-\cos x}\right)^{x^{7/x}}$$

$$= e \qquad =$$

$$x^{7/x} \cdot \log\left(\frac{3x^2+1}{x-\cos x}\right)$$

$$= e \qquad =$$

$$e^{\log x^{7/x}} \cdot \log\left(\frac{3x^2+1}{x-\cos x}\right)$$

$$= e \qquad =$$

$$\left\{ e^{\left(\frac{7}{x} \cdot \log x\right)} \cdot \log\left\{\frac{\cancel{x^2}\left(3+\overset{\to 0}{\frac{1}{x^2}}\right)}{\cancel{x}\left(1-\underset{\to 0}{\frac{\cos x}{x}}\right)}\right\}\right\}$$

$$= e$$

$(3x^2+1) \sim 3x^2$ per $x \to +\infty$

$(x-\cos x) \simeq x$ per $x \to +\infty$

- - - - 1ª parte   2 parte

$$\lim_{x\to+\infty}\left(\qquad\qquad \right) \cdot \lim_{x\to+\infty}\left(e^{e^{\left(\frac{7}{x}\cdot\log x\right)}\cdot\log 3x}\right)$$

$x \to +\infty$

$$\frac{\log x}{x} \rightsquigarrow 0$$

$$e^{7\cdot\frac{\log x}{x}} \rightsquigarrow 1$$

$$\simeq e^{\underbrace{e^{\overset{\rightarrow 0}{\frac{7}{x}\log x}}}_{\rightsquigarrow 1}\cdot\log 3x} \simeq e^{\log 3x}$$

Insieme le 2 parti'

$$= \lim_{x \to +\infty} \frac{3x}{x^{\frac{1}{2}}\left(2x^{\frac{1}{2}}\right)(2x)} \cdot e^{\log 3x} =$$

$$= \lim_{x \to +\infty} \frac{3x}{4x^{2}} \cdot 3x =$$

$$= \frac{9}{4} \; !$$

$$\underset{x\to 0^+}{\ell}\ \frac{1-\sin x-\cos^2 x}{e^{x^2}-1} =$$

$$= \underset{x\to 0^+}{\ell}\ \frac{1-\sin x-\cos^2 x}{x^2}\cdot\frac{1}{\underbrace{\frac{e^{x^2}-1}{x^2}}_{\to 1}} =$$

$$= \underset{x\to 0^+}{\ell}\left(\underbrace{\frac{1-\cos^2 x}{x^2}}_{\to 1} - \frac{\sin x}{x^2}\right) =$$

$$\frac{\sin x}{x^2} \to \underbrace{\frac{\sin x}{x}}_{\to 1}\cdot\underbrace{\frac{1}{x}}_{\to +\infty}$$

$$= -(+\infty) = -\infty$$

Oppure $\dfrac{1-\cos^2 x-\sin x}{x^2} = \dfrac{\sin^2 x-\sin x}{x^2} =$

$= \dots$

$$\lim_{x\to\frac{\pi}{2}} \left[1+\sin\left(\frac{\pi}{2}-x\right)\right]^{\frac{2}{\sin\left(\frac{\pi}{2}-x\right)}} \cdot$$

$$\cdot \ln\left[3\,\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\left(\frac{\pi}{2}-x\right)^2}\right]$$

$$\frac{\pi}{2}-\underset{\downarrow\,\pi/2}{x}=\underset{\downarrow\,0}{t}$$

$$\lim_{t\to 0}\left(1+\sin t\right)^{\frac{2}{\sin t}} \cdot \log\left(3\,\underbrace{\frac{1-\cos t}{t^2}}_{\to 1/2}\right)$$

$$\underbrace{\sin \underset{\downarrow\,0}{t}}_{\downarrow\,0} = \frac{1}{p_{\to +\infty}}$$

$$= \ln\frac{3}{2} \cdot \lim_{p\to+\infty}\left(1+\frac{1}{p}\right)^{2p} =$$

$$= \ln\frac{3}{2}\,\lim_{p\to+\infty}\left(\underbrace{\left(1+\frac{1}{p}\right)^{p}}_{\to e}\right)^{2} = e^{2}\cdot \ln\frac{3}{2}$$

$$\cdot \lim_{x\to+\infty} \frac{\ln(1+2^x)}{\ln^2 x}\left[1-\cos\left(\frac{\ln x}{\sqrt{x}}\right)\right] =$$

$$\Rightarrow \quad \frac{\ln x}{\sqrt{x}} \to 0 \quad \text{per } x \to +\infty$$

$$\cos 0 \to 1$$

$$1-\cos(\qquad) \to 0$$

$$= \lim_{x\to+\infty} \frac{\ln\left[2^x\left(\frac{1}{2^x}+1\right)\right]}{\ln^2 x} \cdot \left(\frac{\ln x}{\sqrt{x}}\right)^2 \cdot \underbrace{\frac{1-\cos\left(\frac{\ln x}{\sqrt{x}}\right)}{\left(\frac{\ln x}{\sqrt{x}}\right)^2}}_{1/2} =$$

$$\simeq \frac{1}{2} \, \frac{\ln 2^x + \cancel{\ln\left(\overset{\to 0}{\frac{1}{2^x}}+\overset{1}{1}\right)}}{\cancel{\ln^2 x}} \cdot \frac{\cancel{\ln^2 x}}{x} =$$

$$= \frac{1}{2} \, \frac{\cancel{x}\cdot \ln 2}{\cancel{x}} = \frac{1}{2} \ln 2$$

RICORDA

$$\lim_{x \to 0} \frac{\sinh x}{x} = 1$$

$$\lim_{f(x) \to 0} \frac{\sinh(f(x))}{f(x)} = 1$$

$$\lim_{x \to 0} \frac{\cosh x - 1}{x^2} = \frac{1}{2}$$

$$\lim_{f(x) \to 0} \frac{\cosh f(x) - 1}{f^2(x)} = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\tanh x}{x} = 1$$

$$\lim_{f(x) \to 0} \frac{\tanh f(x)}{f(x)} = 1$$

$$\lim_{x \to -\infty} \frac{x^5 - (\sinh x)^2}{(\ln|x|)^{10} + e^{-2x}}$$

$$\cdot \lim_{x \to 0} \frac{\sqrt[5]{(1+x^2)} - 1}{\arcsin^2 x}$$

$$\cdot \lim_{x \to 0} \frac{\arctan(\overset{\to 0}{\sinh x})}{x} =$$

$$= \lim_{x \to 0} \underbrace{\frac{\arctan(\sinh x)}{\sinh x}}_{\downarrow 1} \cdot \underbrace{\frac{\sinh x}{x}}_{\downarrow 1} =$$

$$= 1$$

$$\cdot \lim_{x\to+\infty} \frac{8^{x} + 9^{\ln x} + 7x^{10}}{(\ln x)^8 + 2^{3x+1}} =$$

$$= \lim_{x\to+\infty} \frac{2^{3x} + 9^{\ln x} + 7x^{10}}{\left(\frac{\ln^8 x}{2^{3x+1}} + 1\right) 2^{3x+1}} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{2^{3x}}\left[1 + \overset{\to 0}{\frac{9^{\ln x}}{2^{3x}}} + \overset{\to 0}{\frac{7x^{10}}{2^{3x}}}\right]}{\cancel{2^{3x}}\cdot 2\left(\frac{\ln^8 x}{2^{3x}\cdot 2} + 1\right)} =$$

$$\frac{9^{\ln x}}{2^{3x}} = \frac{e^{\log 9^{\ln x}}}{e^{\log 2^{3x}}} = e^{\ln x\cdot\ln 9 - 3x\cdot\ln 2} =$$

$$= e^{\underset{-\infty}{\underset{\downarrow}{-3x}}\left(\underbrace{\frac{\ln x\cdot\overset{\approx 2}{\ln 9}}{3x}}_{\downarrow 0} - \overset{>0}{\ln 2}\right)} \to 0$$

$$= \frac{1}{2}$$

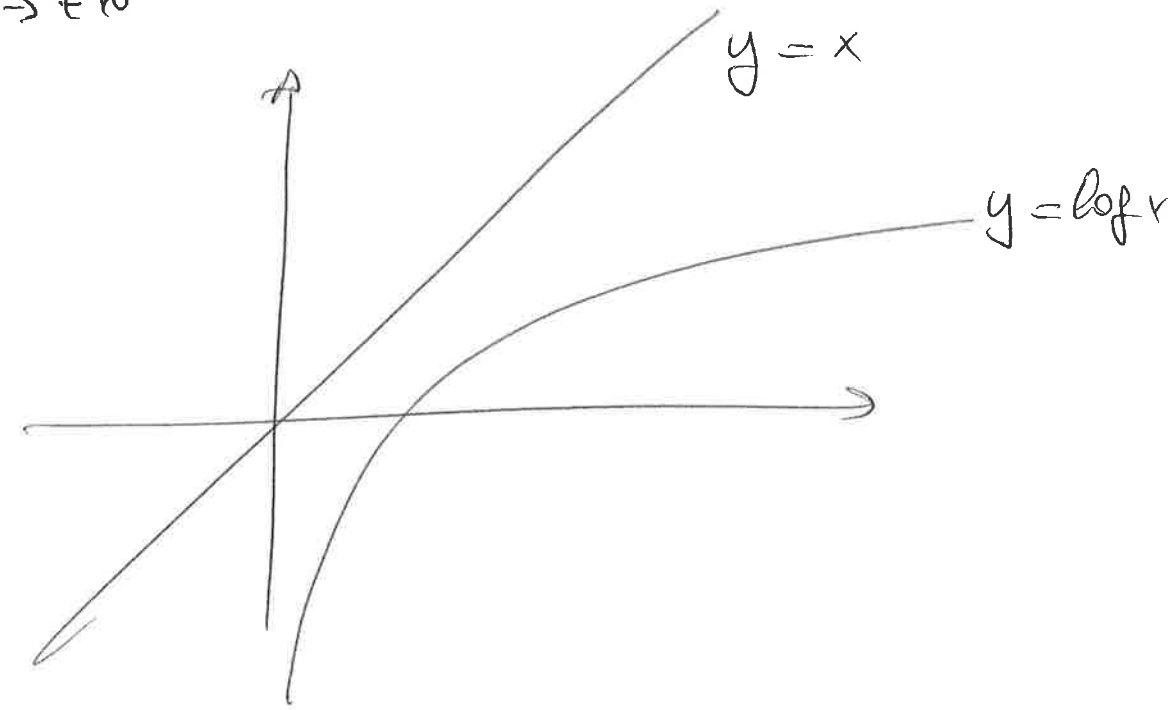
$$\lim_{x \to +\infty} \left(\frac{\sqrt[4]{x^2+3x+1} - \sqrt{x}}{\sqrt{x} + 7\ln^2 x} \right) \cdot \left(\frac{3x^2+1}{x - \cos(x)} \right) \cdot x^{7/x}$$

per

alle

① $$x^{7/x} = e^{\log x^{7/x}} = e^{\frac{7}{x} \cdot \log x} = e^{7 \cdot \frac{\log x}{x}}$$

$$\lim_{x \to +\infty} \frac{\log x}{x} = 0$$



Confronto fra infiniti

$$\boxed{e^{0 \cdot 7} = 1}$$

② $\frac{3x^2+1}{x - \cos x} \sim 3x$

$-1 \le \cos x \le 1$

$x+1 \le x - \cos x \le x - 1$

$$\begin{array}{ccc} x-1 & \le x - \cos x \le & x+1 \\ \downarrow & & \downarrow \\ +\infty & & +\infty \end{array}$$

$\frac{3x^2+1}{x\left[1 - \frac{\cos x}{x}\right]} \simeq \frac{3x^2+1}{x} \simeq \boxed{3x}$

$-1 \le \cos x \le 1$

$$\begin{array}{ccc} -\frac{1}{x} & \le \frac{\cos x}{x} \le & 1/x \\ \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$

Teo Carabinieri

$x \to \infty$

③

$$\left(\sqrt[4]{x^2+3x+1}-\sqrt{x}\right) = +\infty - \infty \quad \text{F.I}$$

$$\frac{\left(\sqrt[4]{x^2+3x+1}-\sqrt{x}\right)\left(\sqrt[4]{x^2+3x+1}+\sqrt{x}\right)}{\sqrt[4]{x^2+3x+1}+\sqrt{x}}$$

$$= \frac{\sqrt[\overset{2}{\cancel{4}}]{\left(x^2+3x+1\right)^2}-x}{\sqrt[4]{x^2+3x+1}+\sqrt{x}} \cdot \frac{\sqrt{x^2+3x+1}+x}{\sqrt{x^2+3x+1}+x}$$

$$= \frac{\cancel{x^2}+3x+1\cancel{-x^2}}{\left(\underset{+\infty}{\sqrt[4]{x^2+3x+1}}+\underset{+\infty}{\sqrt{x}}\right)\left(\underset{+\infty}{\sqrt{x^2+3x+1}}+\underset{+\infty}{x}\right)} =$$

$$\sqrt{x^2\left(1+\frac{3}{x}+\frac{1}{x^2}\right)}+x$$

$$= \frac{3x+1}{x^{2/4}\left(\sqrt[4]{1+\frac{3}{x}+\frac{1}{x^2}}+1\right) \cdot x\left(\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+1\right)} =$$

RACC. FORZATO

$$x^{2/4} = x^{1/2} = \sqrt{x}$$

$$= \frac{3x+1}{x^{3/2}\left(\sqrt[4]{\cancel{1}+\frac{3x}{x}+\frac{1}{x^2}}+1\right)\left(\sqrt{1+\frac{3}{x}+\frac{1}{x^2}}+1\right)}$$

$$(1+1)\underbrace{(1+1)}_{\text{Trascurabile}} = 4$$

$$\frac{\sqrt[4]{x^2+3x+1}-\sqrt{x}}{(\underset{+\infty}{\sqrt{x}}+7\underset{+\infty}{\ln^2 x})} = \frac{3x+1}{4x^{3/2}x^{1/2}\left(1+\frac{7\ln^2 x}{\sqrt{x}}\right)}$$

$$\frac{\ln^2 x}{\sqrt{x}}$$