SERIE

$\boxed{\text{C.N.}}$

$$\boxed{\sum_{n=n_0}^{\infty} a_n \text{ converge} \Rightarrow \lim_{n\to+\infty} a_n = 0}$$

infinitesimo

se $\lim_{n\to+\infty} a_n \neq 0 \Rightarrow$ divergente $\sum_{n=n_0}^{+\infty} a_n$

se $\lim_{n\to+\infty} a_n = 0 \not\Rightarrow$

CONTROESEMPIO

$$\lim_{n\to+\infty} \frac{1}{n} = 0$$

$\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ non è convergente

$$\sum_{n=0}^{\infty} (-3n^2 - 2n + 6) =$$

$$\lim_{n \to +\infty} (-3n^2 - 2n + 6) = -\infty$$

$$\sum_{n=0}^{\infty} \frac{n^2+3}{2n^2+4} = \qquad \text{S. T. P.}$$

$$\lim_{n \to +\infty} \frac{n^2+3}{2n^2+4} = \frac{1}{2}$$

$$\sum_{n=1}^{\infty} \frac{n-3}{n^3-4}$$

$$\lim_{n \to +\infty} \frac{n-3}{n^3-4} \simeq \lim_{n \to +\infty} \frac{\cancel{n}}{n^{3\,2}} = 0$$

SERIE

- TERMINI POSITIVI

$$\sum_{n=0}^{+\infty} \frac{(n^2+4n+1)}{(n^2+4)\, e^n}$$

- TERMINI NEGATIVI

$$\sum_{n=1}^{+\infty} \left(-\frac{4}{n^2}\right) = -4 \underbrace{\sum_{n=1}^{+\infty} \frac{1}{n^2}}_{\text{S.T.P.}}$$

- SEGNO ALTERNO

$$\sum_{n=2}^{+\infty} (-1)^n \frac{n^2+1}{\underbrace{n^3+4}_{+}}$$

PRINCIPALI SERIE

SERIE GEOMETRICA

$$\sum_{n=1}^{\infty} q^n$$

$$S_n = \frac{1-q^{n-1}}{1-q}$$

$$S = \lim_{n\to+\infty} S_n = \lim_{n\to+\infty} \frac{1-q^{n-1}}{1-q} = \begin{cases} \frac{1}{1-q} & |q|<1 \\ +\infty & q \geq 1 \\ \nexists & q \leq -1 \end{cases}$$

q = Ragione della serie geometrica

$$\sum_{n=0}^{\infty} \left(\frac{x^2-1}{x^2-2}\right)^n$$

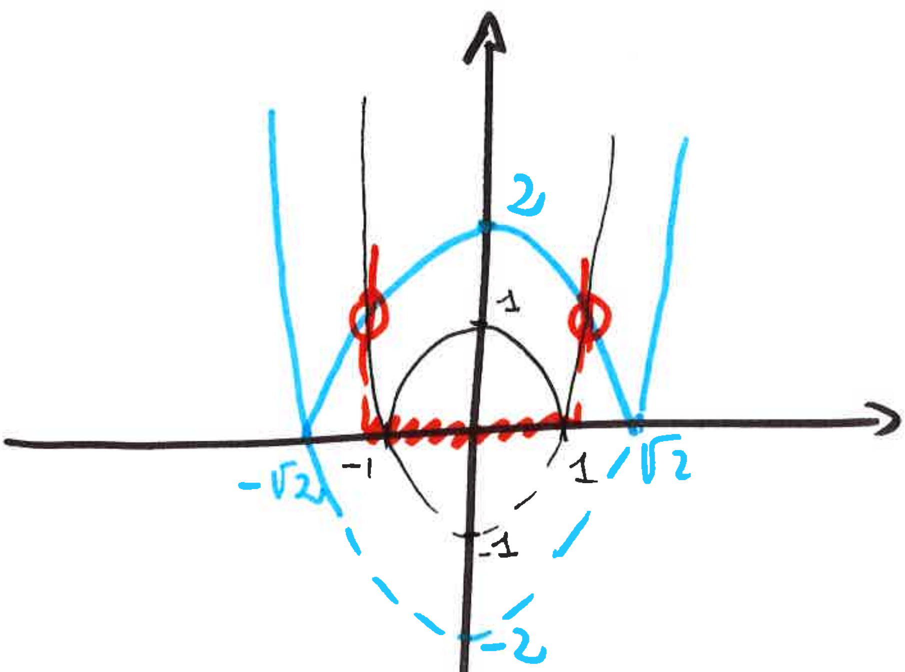
convergenza $\iff$ |Ragione| < 1

$$\left|\frac{x^2-1}{x^2-2}\right| < 1$$

$$|x^2-1| < |x^2-2| \qquad \left(\begin{array}{l} |x^2-2| > 0 \\ x \neq \pm\sqrt{2} \end{array}\right)$$

$y = |x^2-1|$

$y = |x^2-2|$



prima parabola $<$ seconda parabola

$$y = x^2 - 1$$

$$y = -x^2 + 2$$

$$x^2 - 1 = -x^2 + 2$$

$$2x^2 = 3$$

$$x = \pm\sqrt{3/2}$$

soluzione cercata

$$-\sqrt{3/2} < x < \sqrt{3/2}$$

$$S = \frac{1}{1-q} = \frac{1}{1 - \frac{x^2-1}{x^2-2}} = \frac{x^2-2}{x^2-2-x^2+1} =$$

$$= 2 - x^2$$

SERIE ARMONICA

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

nonostante

$$\lim_{n \to +\infty} \frac{1}{n} = 0$$

(infinitesimo)

essa non converge

Serie armonica generalizzata

$$\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}$$

$\alpha > 1$

convergente

se $\alpha \leq 1$ divergente

SERIE TELESCOPICA (MENGOLI

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

$$a_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\frac{A}{n} - \frac{B}{n+1} =$$

$$= \frac{A(n+1) - B \cdot n}{n(n+1)} =$$

$$= \frac{(A-B)n + A}{n(n+1)} =$$

$$\begin{cases} A-B=0 \\ A=1 \end{cases} \qquad \begin{cases} A=1 \\ B=+A=+1 \end{cases}$$

$$S_n = \underbrace{\left(1 - \cancel{\frac{1}{2}}\right)}_{n=1} + \underbrace{\left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right)}_{n=2} + \underbrace{\left(\cancel{\frac{1}{3}} - \frac{1}{4}\right)}_{n=3} + \ldots$$

$$\ldots + \underbrace{\left(\frac{1}{n} - \frac{1}{n+1}\right)}_{n} = 1 - \frac{1}{n+1}$$

$$a_n = \frac{1}{n} - \frac{1}{n+1} = \left(\lim_{n \to +\infty} a_n = 0\right)$$

$$S_n = 1 - \frac{1}{n+1}$$

$$\lim_{n \to +\infty} S_n = \lim_{n \to +\infty} \left(1 - \frac{1}{n+1}\right) = 1$$

serie convergente

$$\begin{cases} \cancel{3}\cdot(A-B)=0 \\ 5A-2B=1 \end{cases}$$

$$\begin{cases} {}^{2}A-{}^{2}B=0 \\ 5A-2B=1 \end{cases}$$

$$\begin{cases} -3A=-1 \\ B=A \end{cases} \qquad \begin{cases} A=\frac{1}{3} \\ B=\frac{1}{3} \end{cases}$$

$$a_n=\frac{1}{(3n+2)(3n+5)}=\frac{1/3}{3n+2}-\frac{1/3}{3n+5}$$

$$\sum_{n=1}^{+\infty}\frac{1}{(3n+2)(3n+5)}=\sum_{n=1}^{+\infty}\left\{\frac{1/3}{3n+2}-\frac{1/3}{3n+5}\right\}$$

$$\sum_{n=1}^{\infty} \frac{1}{(3n+2)(3n+5)}$$

$$a_n = \frac{1}{(3n+2)(3n+5)} =$$

$$\frac{A}{3n+2} - \frac{B}{3n+5} =$$

$$= \frac{A(3n+5) - B(3n+2)}{(3n+2)(3n+5)} =$$

$$= \frac{A \cdot 3n + 5A - B \cdot 3n - 2B}{(3n+2)(3n+5)} =$$

$$= \frac{3n(A-B) + (5A-2B)}{(3n+2)(3n+5)}$$

$$= \frac{1}{3} \sum_{n=1}^{+\infty} \left\{ \frac{1}{3n+2} - \frac{1}{3n+5} \right\}$$

$$S_n = \frac{1}{3} \left\{ \underbrace{\left(\frac{1}{5} - \cancel{\frac{1}{8}} \right)}_{n=1} + \underbrace{\left(\cancel{\frac{1}{8}} - \cancel{\frac{1}{11}} \right)}_{n=2} + \underbrace{\left(\cancel{\frac{1}{11}} - \cancel{\frac{1}{14}} \right)}_{n=3} + \right.$$

$$\left. + \cdots + \underbrace{\left(\cancel{\frac{1}{3n+2}} - \frac{1}{3n+5} \right)}_{n} \right\} =$$

$$= \frac{1}{3} \left\{ \frac{1}{5} - \frac{1}{3n+5} \right\}$$

$$\lim_{n\to+\infty} \frac{1}{3} \left\{ \frac{1}{5} - \underbrace{\frac{1}{3n+5}}_{0} \right\} = \frac{1}{15}$$

convergente

CRITERI PER SERIE A TERMINI POSITIVI

- **CONFRONTO**

$\sum_n a_n \qquad \sum_n b_n \qquad a_n \leq b_n$

$\sum_n b_n$ conv $\Rightarrow \sum_n a_n$ conv.

$\sum_n a_n$ div. $\Rightarrow \sum_n b_n$ div.

- **CONFRONTO ASINTOTICO**

$$l = \lim_{n \to +\infty} \frac{a_n}{b_n} \quad \begin{cases} l \in]0, +\infty[& \sum_n a_n \text{ conv.} \Leftrightarrow \sum b_n \text{ conv.} \\ l = 0 & \sum b_n \text{ con} \Rightarrow \sum a_n \text{ con.} \\ l = +\infty & \sum b_n \text{ div.} \Rightarrow \sum a_n \text{ div.} \end{cases}$$

- **RADICE**

$$l = \lim_{n \to +\infty} \sqrt[n]{a_n} \quad \begin{cases} l < 1 & \sum_n a_n \text{ conv.} \\ l > 1 & \sum_n a_n \text{ div.} \\ l = 1 & \text{nulla} \end{cases}$$

- **QUOZIENTE / RAPPORTO**

$$l = \lim_{n \to +\infty} \frac{a_{n+1}}{a_n} \quad \begin{cases} l < 1 & \sum_n a_n \text{ conv.} \\ l > 1 & \sum_n a_n \text{ diverge} \\ l = 1 & \text{nulla} \end{cases}$$

Per confrontare le serie:

- $\sum_{n=0}^{+\infty} q^n$ serie geometrica

- $\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ serie armonica gen.

  $\alpha > 1$ conv.

  $\alpha \le 1$ diverg.

- $\sum_{n=2}^{+\infty} \frac{1}{n^\alpha (\log n)^\beta}$ $\alpha, \beta \in \mathbb{R}$

conv. se $\alpha > 1$, $\forall \beta$

se $\alpha = 1$ $\beta > 1$

diverg. positive

se $\alpha < 1$ $\forall \beta$

se $\alpha = 1$ $\beta \le 1$

$$\cdot \sum_{n=2}^{+\infty} \frac{1}{e^{\gamma n} \cdot n^{\alpha} \cdot (\log n)^{\beta}} \qquad \alpha, \beta, \gamma \in \mathbb{R}$$

se $\gamma > 0$

le serie conv. $\forall \alpha, \beta$

se $\gamma < 0$

le serie diverge $\forall \alpha, \beta$

se $\gamma = 0$

ci si ritrova nel caso precedente

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{n^3(2n)!}$$

$$\frac{a_{n+1}}{a_n}$$

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} =$$

$$a_{n+1} = \frac{\left((n+1)!\right)^2}{(n+1)^3 \underbrace{\left(2(n+1)\right)!}_{(2n+2)!}}$$

$$= \lim_{n\to+\infty} \underbrace{\frac{\left((n+1)!\right)^2}{(n+1)^3 \cdot (2n+2)!}}_{a(n+1)} : \underbrace{\frac{(n!)^2}{n^3(2n)!}}_{a_n} =$$

$$\lim_{n\to+\infty} \frac{\left((n+1)!\right)^2}{(n+1)^3 (2n+2)!} \cdot \frac{n^3 (2n)!}{(n!)^2} =$$

$$= \lim_{n \to +\infty} \frac{(n+1)^2 \cdot \cancel{(n!)^2}}{(n+1)^3 (2n+2)(2n+1) \cancel{(2n)!}} \cdot \frac{n^3 \cancel{(2n)!}}{\cancel{(n!)^2}} =$$

= grado num: $\rightsquigarrow 5$

grado den $\longrightarrow 5$

$$= \lim_{n \to +\infty} \frac{n^3}{(n+1)^3} \cdot \frac{(n+1)^2}{(2n+2)(2n+1)} =$$

$$= \lim_{n \to +\infty} \frac{n^3}{n^3} \cdot \frac{n^2}{(2n)^2} = \frac{1}{4}$$

$$\bullet \quad \sum_{n=1}^{\infty} \frac{n^3+\sin n}{2n^5+\log n+n}$$

c. q.

$$a_n = \frac{n^3+\sin n}{2n^5+\log n+n} =$$

$$= \frac{n^3\left(1+\frac{\sin n}{n^3}\right)}{n^5\left(2+\frac{\log n}{n^5}+\frac{n}{n^5}\right)}$$

$n \to +\infty$

$$\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \frac{n^3+\sin n}{2n^5+\log n+n} =$$

$$= \lim_{n\to+\infty} \frac{n^3\left(1+\frac{\sin n}{n^3}\right)}{n^5\left(2+\frac{\log n}{n^5}+\frac{n}{n^5}\right)} =$$

$$= \lim_{n\to+\infty} \frac{1}{2\cdot n^2} = 0$$

. S. T. P.

. succ. a_n è infinitesima

C. C. A.

$n \to +\infty$

$NUM = n^3 + \sin n \sim n^3$

$DEN = 2n^5 + \log n + n \sim 2n^5$

Serie di confronto

$$\sum_{n=1}^{+\infty} \frac{1}{2n^2} = \frac{1}{2} \underbrace{\sum_{n=1}^{+\infty} \frac{1}{n^2}}$$

serie armonica generalizzata

$\alpha = 2 > 1$

conv.

$\Downarrow$

$a_n < b_n = \frac{1}{2} \frac{1}{n^2}$

$$\sum \frac{n^3 + \sin n}{2n^5 + \log n + n} \text{ conv}$$

$$\bullet \sum_{n=0}^{+\infty} \frac{n+3}{(n+2)\sqrt{n+4}}$$

C. C.

S. T. P.

$$\lim_{n\to+\infty} \frac{n+3}{(n+2)(\sqrt{n+4})} \simeq$$

$$\simeq \lim_{n\to+\infty} \frac{\cancel{n}}{\cancel{n}\cdot n^{1/2}} = 0$$

a_n è infinitesimo

Ricerca di una successione da confrontare

$$\frac{n+3}{(n+2)(\sqrt{n+4})} > \frac{n+3}{(n+2)(n+4)} >$$

$$> \frac{\cancel{n+3}}{(\cancel{n+3})(n+4)} = \frac{1}{n+4} \sim \frac{1}{n}$$

$\sum \frac{1}{n}$ serie armonica divergente.

$$a_n = \frac{n+3}{(n+2)\sqrt{n+4}} > \underbrace{b_n = \frac{1}{n+4} \sim \frac{1}{n}}$$

$$\sum a_n \qquad \Leftarrow \qquad \sum b_n$$

divergente $\qquad\qquad$ divergente

N.B.

$$\sum_{n=1}^{+\infty} \frac{1}{n+4} = \underbrace{\sum_{n=1}^{+\infty} \frac{1}{n}}_{\text{diverge}} - \frac{1}{2} - \frac{1}{3} - \frac{1}{4} - 1$$

$$\cdot \sum_n \frac{n}{2n^2-1}$$

S.T.P.

$$\lim_{n\to+\infty} \frac{n}{2n^2-1} \simeq \lim_{n\to+\infty} \frac{\cancel{n}}{2n^{\cancel{2}}} = 0$$

$$a_n = \frac{n}{2n^2-1} > \frac{n}{2n^2} = \frac{1}{2n}$$

$$2n^2-1 < 2n^2$$

$$\sum \frac{1}{2n} = \frac{1}{2} \sum n$$

serie armonica divergente

$\sum a_n$ divergente

- $\sum_n \frac{n}{(n+2)^3}$

S.T.P.

$$\lim_{n\to+\infty} \frac{n}{(n+2)^3} \simeq \lim_{n\to\infty} \frac{n}{n^3} =$$

$$= \lim_{n\to+\infty} \frac{1}{n^2} = 0$$

catene del confronto

$$\frac{n}{(n+2)^3} < \frac{n}{n^3} = \frac{1}{n^2}$$

$$n+2 > n$$

$\sum \frac{1}{n^2}$ serie arm. generale.

$\alpha = 2$

$\Downarrow$

$\sum a_n$ conv. $\Leftarrow$ conv

$$\sum_{n=2}^{\infty} \frac{2^{-2n+1}}{3^{n-2}}$$

$$a_n = \frac{2^{-2n+1}}{3^{n-2}} = \frac{2^{-2n} \cdot 2}{3^n \cdot 3^{-2}} = \frac{(2^{-2})^n \cdot 2}{3^n \cdot 3^{-2}} =$$

$$= \left(\frac{1}{4 \cdot 3}\right)^n \cdot 2 \cdot 9 = 18 \cdot \left(\frac{1}{12}\right)^n$$

$$\sum_{n=2}^{+\infty} \frac{2^{-2n+1}}{3^{n-2}} = \sum_{n=2}^{+\infty} 18 \cdot \left(\frac{1}{12}\right)^n =$$

$$= 18 \cdot \sum_{n=2}^{+\infty} \left(\frac{1}{12}\right)^n$$

- $\sum_{n=2}^{+\infty} \left(\frac{1}{12}\right)^n$ serie geometrica

$$q = \frac{1}{12} \qquad |q| < 1$$

$\Rightarrow$ convergente

$$S = 18 \left(\frac{1}{1-q} - q^0 - q^1 \right) = \qquad n = 2$$

$$= 18 \left(\frac{1}{1-\frac{1}{12}} - 1 - \frac{1}{12} \right) =$$

$$= \dots = \frac{3}{22}$$

$$\cdot \sum_{n=1}^{\infty} \frac{n^5}{e^{n}} \left[\log\left(1+\frac{1}{e^n}\right)^{e^n}\right]^{n}$$

$\sqrt[n]{a^n}$

$$\lim_{n\to+\infty} \sqrt[\cancel{n}]{\frac{n^{5/n}}{e^{\cancel{n}}} \cdot \left[\log\left(1+\frac{1}{e^n}\right)^{e^n}\right]^{\cancel{n}}} =$$

$$= \lim_{n\to+\infty} \frac{n^{5/n}}{e} \left[\log\left(1+\frac{1}{e^n}\right)^{e^n}\right] = \frac{1}{e} < 1$$

CONV.

$$\lim_{n\to+\infty} n^{5/n} = \lim_{n\to+\infty} e^{\log n^{5/n}} =$$

$$= \lim_{n\to+\infty} e^{\frac{5\log n}{n}} \rightarrow 0 \quad \rightarrow e^0 = 1$$

$$\lim_{n\to+\infty} \log\left(1+\frac{1}{e^n}\right)^{e^n} = \qquad e^n = t$$

$$= \lim_{n\to+\infty} \log\left(1+\frac{1}{t}\right)^t = \log e = 1$$

$$\cdot \sum_{n=1}^{+\infty} \frac{2^n}{3^{n+1}} \left(1+\frac{1}{n}\right)^{n^2} =$$

$$\lim_{n\to+\infty} \sqrt[n]{\frac{2^n}{3^{n+1}} \left(1+\frac{1}{n}\right)^{n^2}} =$$

$$= \lim_{n\to+\infty} \frac{2}{3^{\frac{n+1}{n}}} \cdot \left(1+\frac{1}{n}\right)^n$$

$$= \lim_{n\to+\infty} \frac{2}{3^{1+\frac{1}{n}\to 0}} \underbrace{\left(1+\frac{1}{n}\right)^n}_{e} \cdot$$

$$= \frac{2}{3} e > 1 \quad \text{non conv.}$$

$$\cdot \sum_{n=1}^{+\infty} \left(\frac{n^3-1}{n^3+1}\right)^{n^4}$$

$$\lim_{n\to+\infty} \sqrt[n]{\left(\frac{n^3-1}{n^3+1}\right)^{n^4}} =$$

$$= \lim_{n\to+\infty} \left(\frac{n^3-1}{n^3+1}\right)^{n^3} =$$

$$= \lim_{n\to+\infty} \left(\frac{n^3+1-2}{n^3+1}\right)^{n^3} =$$

$$= \lim_{n\to+\infty} \left(\frac{n^3+1}{n^3+1} - \frac{2}{n^3+1}\right)^{n^3} =$$

$$= \lim_{n\to+\infty} \left(1 + \frac{1}{-\frac{n^3+1}{2}}\right)^{n^3} =$$

$$- \frac{n^3+1}{2}$$

$$\frac{n^3+1-1}{2} \cdot 2 =$$

$$= \left(\frac{n^3+1}{2} - \frac{1}{2}\right)^{(+2)} =$$

$$= \left(- \frac{n^3+1}{2} + \frac{1}{2}\right)^{(-2)}$$

$$t = - \frac{n^3+1}{2} \qquad t \to +\infty \quad n \to +\infty$$

$$n^3 = 1-2t$$

$$= \lim_{t \to +\infty} \left(1 + \frac{1}{t}\right)^{-2t+1} =$$

$$= \lim_{t \to +\infty} \underbrace{\left(1+\frac{1}{t}\right)^{-2t}}_{\to e^{-2}} \cdot \underbrace{\left(1+\frac{1}{t}\right)^{1}}_{\to 1} = e^{-2} < 1$$

conv.

$$\sum_{n=1}^{+\infty} \frac{(n+1)^2}{(n+2)!}$$

$$\lim_{n\to+\infty} \frac{((n+1)+1)^2}{((n+1)+2)!} \cdot \frac{(n+2)!}{(n+1)^2} =$$

$$= \lim_{n\to+\infty} \frac{(n+2)^2}{(n+3)!} \cdot \frac{(n+2)!}{(n+1)^2} =$$

$$= \lim_{n\to+\infty} \frac{(n+2)^2}{(n+1)^2} \cdot \frac{\cancel{(n+2)!}}{(n+3)\cancel{(n+2)!}} =$$

$$= \lim_{n\to+\infty} \frac{n^2}{n^2 \cdot n} = 0$$

conv.

$$\sum_{n=1}^{+\infty} \frac{2^{n+1}}{(n-1)!\; n^n}$$

$$\lim_{n\to+\infty} \frac{2^{(n+1)+1}}{\left((n+1)-1\right)!\,(n+1)^{n+1}} \cdot \frac{(n-1)!\; n^n}{2^{n+1}} =$$

$$= \lim_{n\to+\infty} \frac{2^{n+2}}{2^{n+1}} \cdot \frac{(n-1)!}{n!} \cdot \frac{n^n}{(n+1)^{n+1}} =$$

$$= \lim_{n\to+\infty} 2 \cdot \frac{\cancel{(n-1)}\,!}{n\,\cancel{(n-1)}\,!} \; \frac{n^n}{(n+1)\,(n+1)^n} =$$

$$= \lim_{n\to+\infty} 2 \; \underbrace{\frac{1}{n(n+1)}}_{0} \; \underbrace{\left(\frac{n}{n+1}\right)^n}_{e^{-1}} =$$

$$\left(\frac{n}{n+1}\right)^n \Rightarrow \left(\frac{n+1}{n}\right)^{-n} = \left[\left(1+\frac{1}{n}\right)^n\right]^{-1}$$

$= 0$ conv.

$$\cdot \sum_{n=1}^{+\infty} \frac{n^3 + \log n + 5 \sin n}{5n^7 + 9}$$

$$a_n = \frac{n^3 \left(1 + \frac{\log n}{n^3} + 5 \frac{\sin n}{n^3}\right)}{n^7 \left(5 + \frac{9}{n^7}\right)}$$

con $\frac{\log n}{n^3} \to 0$, $\frac{\sin n}{n^3} \to 0$, $\frac{9}{n^7} \to 0$

$n \to +\infty$

$$a_n \simeq \frac{n^3}{n^7 \cdot 5} = \frac{1}{5} \cdot \frac{1}{n^4} \simeq \frac{1}{n^4}$$

$$\sum a_n \simeq \sum \frac{1}{n^4}$$

serie armonica generalizzata

$\alpha = 4 > 1$

conv.

$$\sum_{n=2}^{+\infty} \frac{-4n^2+\cos^2(3n)+2^n}{n^5-5^n}$$

$2^n \gg -4n^2 \Rightarrow$ Num >0

$+5^n \gg n^5 \Rightarrow$ Den <0

$$= -\sum_{n=2}^{+\infty} \underbrace{\frac{2^n-4n^2+\cos^2(3n)}{5^n-n^5}}_{\text{S.T.P.}}$$

$$b_n = \frac{2^n\left(1-4\frac{n^2}{2^n}^{\to 0}+\frac{\cos^2(3n)}{2^n}^{\to 0}\right)}{5^n\left(1-\frac{n^5}{5^n}\right)} \simeq$$

$$\simeq \frac{2^n}{5^n} = \left(\frac{2}{5}\right)^n$$

"comportamento asintotico"

$$\sum a_n \simeq \sum b_n = \sum\left(\frac{2}{5}\right)^n$$

Serie geometrica

$q = \frac{2}{5} < 1$ conv.

$$\sum_{n=1}^{+\infty} \frac{n^2 + \log^2 n + \underline{3^{n+1}}}{-3n^4 - 2^{-n} + \underline{3^{2n}}}$$

NUM > 0 $\Rightarrow$ S.T.P

DEN: 3^{2n} prevale su tutti

$$a_n = \frac{3^{n+1}\left(\frac{n^2}{3^{n+1}} + \frac{\log^2 n}{3^{n+1}} + 1\right)}{3^{2n}\left(1 - \frac{3n^4}{3^{2n}} - \frac{2^{-n}}{3^{2n}}\right)} \simeq$$

$$\simeq \frac{3^{n+1}}{3^{2n}} = 3^{n+1-2n} = 3^{1-n} =$$

$$= 3 \cdot 3^{-n} = 3\left(\frac{1}{3}\right)^n$$

$$\sum 3\left(\frac{1}{3}\right)^n = 3 \sum \left(\frac{1}{3}\right)^n$$

serie geometrica

$q = \frac{1}{3} < 1$

convergente

$$\sum_{n=1}^{+\infty} \frac{\tilde{n}^3 - \cos 4n}{\log n^2 + \sin(3n) + \underline{n^4}}$$

S.T.P

$$a_n = \frac{n^3\left(1 - \overbrace{\frac{\cos 4n}{n^3}}^{\to 0}\right)}{n^4\left(1 + \underbrace{\frac{\log n^2}{n^4}}_{n^2 \cdot n^2 \to 0} + \underbrace{\frac{\sin 3n}{n^4}}_{\to 0}\right)} \sim$$

$$\sim \frac{n^3}{n^4} = \frac{1}{n}$$

serie confronto $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n}$

divergente
serie armonica

CONVERGENZA ASSOLUTA

$\sum_n |a_n|$ convergenti $\Rightarrow$ $\sum a_n$ convergente semplice

NO

CONVERGENZA SEMPLICE

$\sum_n a_n$ con segno variabile

$$\sum_n \frac{(-1)^n n^2}{\cos n}$$

CRITERIO DI LEIBNIZ

$$\sum_n (-1)^n b_n$$

se
- $\lim_{n \to +\infty} b_n = 0$ b_n infinitesima
- $\{b_n\}$ monotona decrescente
  $(b_{n+1} \leq b_n \quad \forall n \geq \bar{n} \quad n, \bar{n} \in \mathbb{N})$

$\Rightarrow \sum_n (-1)^n b_n$ è convergente

$$\cdot \sum_n (-1)^n \frac{n^2}{6n^4-4}$$

$$\sum |a_n| = \sum_n \frac{n^2}{6n^4-4}$$

consideriamo i termini assoluti

$$b_n = \frac{n^2}{6n^4-4} = \frac{n^2}{n^4\left(6-\frac{4}{n^4}\right)} \simeq$$

$$\simeq \frac{1}{6n^2}$$

$$\sum \frac{1}{6n^2} = \frac{1}{6} \sum \frac{1}{n^2}$$

serie armonica generalizzata $\alpha = 2 > 1$

conv.

$\Rightarrow \sum |a_n|$ conv. ass.

$\Rightarrow \sum a_n$ conv.

$$\cdot \sum_n (-1)^n \frac{(n+2)}{(n^2+4)\sqrt{n+1}}$$

$$\sum_n |a_n| = \sum_n \frac{n+2}{(n^2+4)\sqrt{n+1}} \quad \text{STP}$$

$$|a_n| = \frac{n+2}{(n^2+4)\sqrt{n+1}} =$$

$$= \frac{\cancel{n}\left(1+\frac{2}{n}\right)}{n^{\cancel{2}}\left(1+\frac{4}{n^2}\right) n^{1/2} \sqrt{1+\frac{1}{n}}} \simeq$$

$$\simeq \frac{1}{n^{3/2}}$$

$\sum \frac{1}{n^{3/2}}$ S. a. g. $\alpha = 3/2 > 1$ conv.

$\Rightarrow \sum |a_n|$ conv. ass.

$\Rightarrow \sum a_n$ conv.

$$\bullet \sum_n \frac{(-1)^n}{\sqrt[3]{n+1}}$$

$$a_n = \frac{1}{\sqrt[3]{n+1}}$$

$$\lim_{n\to+\infty} \frac{1}{\sqrt[3]{n+1}} = 0 \qquad \text{infinitesima}$$

$$a_{n+1} < a_n$$

$$\frac{1}{\sqrt[3]{n+1+1}} < \frac{1}{\sqrt[3]{n+1}} \qquad \text{decrescente}$$

$$\Rightarrow \sum_n \frac{(-1)^n}{\sqrt[3]{n+1}} \qquad \text{convergente}$$

valutare la decrescenza

$$a_{n+1} < a_n$$
$$\Updownarrow$$
$$\frac{a_{n+1}}{a_n} < 1$$

$$\frac{\dfrac{1}{\sqrt[3]{n+2}}}{\dfrac{1}{\sqrt[3]{n+1}}} < 1 ?$$

$$= \sqrt[3]{\frac{n+1}{n+2}} < 1 \quad \text{si}$$

$\forall n \in \mathbb{N}.$

$$\cdot \sum_n \frac{(-1)^n}{(n+1)\sqrt{n}}$$

$$\cdot \lim_{n \to +\infty} \frac{1}{(n+1)\sqrt{n}} = 0 \quad \text{infinitesima}$$

$$\cdot \frac{a_{n+1}}{a_n} =$$

$$= \frac{1}{((n+1)+1)\sqrt{n+1}} \cdot \left[(n+1)\sqrt{n}\right] =$$

$$= \frac{(n+1)\sqrt{n}}{(n+2)\sqrt{n+1}} \leq \frac{\cancel{n+2}}{\cancel{n+2}} \cdot \sqrt{\frac{n}{n+1}} =$$

$$= \sqrt{\frac{n}{n+1}} < 1$$

a_n decrescente

$\Rightarrow$ convergenza

$$\bullet \quad \sum_{n=1}^{+\infty} (-1)^{n+1} \frac{2}{3n^2+4} \sin \frac{1}{\sqrt{n+3}}$$

Termini assoluti

$$\sum_{n=1}^{+\infty} \frac{2}{3n^2+4} \sin \frac{1}{\sqrt{n+3}}$$

$$\frac{2}{3n^2+4} \simeq \frac{2}{3n^2}$$

$$\sin \frac{1}{\sqrt{n+3}} = \sin \frac{1}{\sqrt{n}} = \sin \frac{1}{n^{1/2}} \simeq \frac{1}{n^{1/2}}$$

$$a_n \asymp b_n = \frac{2}{3\, n^2} \cdot \frac{1}{n^{1/2}} = \frac{2}{3\ n^{5/2}}$$

$$\sum b_n = \frac{2}{3} \sum \frac{1}{n^{5/2}} \quad \text{s. a. g.} \quad \alpha = \frac{5}{2} > 1$$

Conv.

$\sum a_n$ converge ass $\to$ conv.

• $\sum_{n=3}^{+\infty} \left(1-\frac{3}{n+1}\right)^{\frac{n^4+2}{n^2-1}}$

S.T.P.

$$\lim_{n\to+\infty} \left(1-\frac{3}{n+1}\right)^{\frac{n^4+2}{n^2-1}} =$$

$$= \lim_{n\to+\infty} \Bigg[\underbrace{\left(1-\frac{3}{n+1}\right)^{\frac{n+1}{3}}}_{\left[e^{-3}\right]^{+\infty}}\Bigg]^{\frac{3}{n+1}\cdot\frac{n^4+2}{n^2-1}} \qquad \frac{3}{n+1}\cdot\frac{n^4+2}{n^2-1} \to \sim \frac{n^4}{n^3} = n$$

$$= e^{-\infty} = 0$$

infinitesimo

C.R.

$$\lim_{n\to+\infty} \sqrt[n]{\left(1-\frac{3}{n+1}\right)^{\frac{n^4+2}{n^2-1}}} =$$

tch -

$$= \lim_{n\to+\infty} \left(1-\frac{3}{n+1}\right)^{\frac{n^4+2}{n(n^2-1)}} =$$

$$= \lim_{n\to+\infty} \Bigg[\underbrace{\left(1-\frac{3}{n+1}\right)^{\frac{n+1}{3}}}_{e^{-3}}\Bigg]^{\underbrace{\frac{3}{n+1}\cdot\frac{n^4+2}{n(n^2-1)}}}$$

$$\frac{3\,(n^4+2)}{(n+1)\,n\,(n^2-1)} \simeq 3\,\frac{n^4}{n^4} = 3$$

$$= \left(e^{-3}\right)^3 = e^{-9} \ll 1$$

convergenze.