EA

Lezione

Stato di funzione

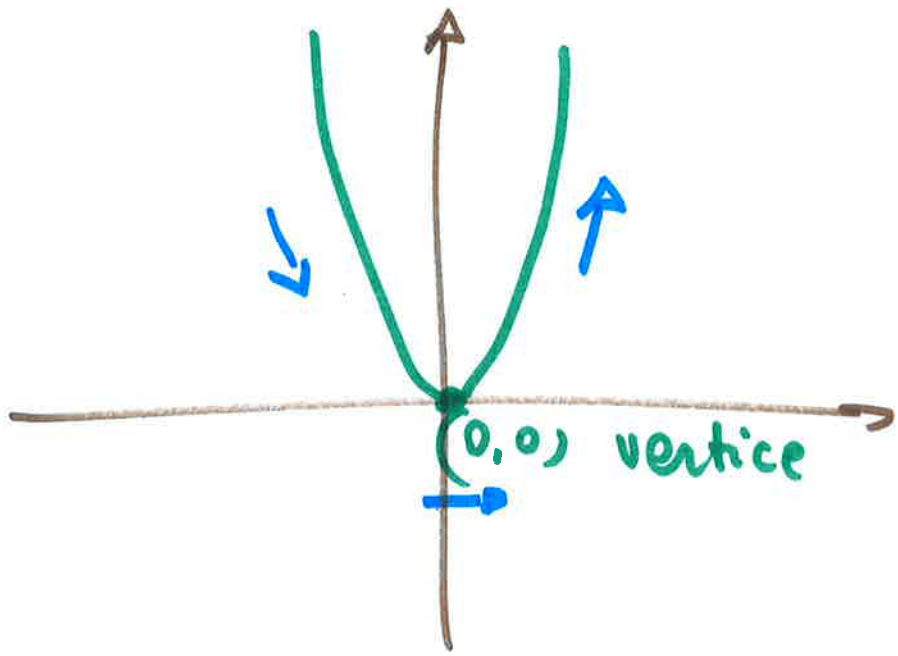
del 27/11/18

STUDIO DI FUNZIONE

- dominio
- simmetrie $\quad f(-x) = \begin{cases} -f(x) \\ f(x) \end{cases}$
- segno $\quad f(x) \geq 0$
- limiti
  → ASINTOTI
  VERTICALE
  ORIZZONTALE
  OBLIQUO
- INTERSEZIONI CON GLI ASSI
  · CONTINUITA'
- DERIVATA PRIMA
  · DOMINIO
  · SEGNO
  · ANDAMENTO
  · MASSIMO/I / MINIMO/I
  · PUNTI DI NON DERIVABILITA'
- DERIVATA SECONDA
  · CONCAVITA' / CONVESSITA'
  · FLESSI

$y = x^2$ parabole

dom $\mathbb{R}$

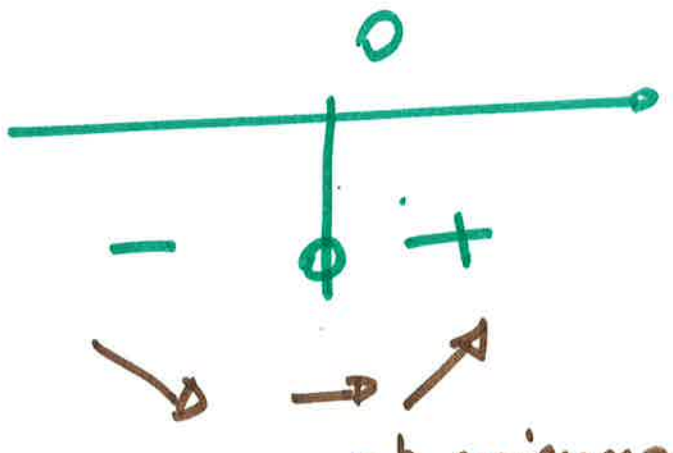


$y' = 2x$

dom $\mathbb{R}$

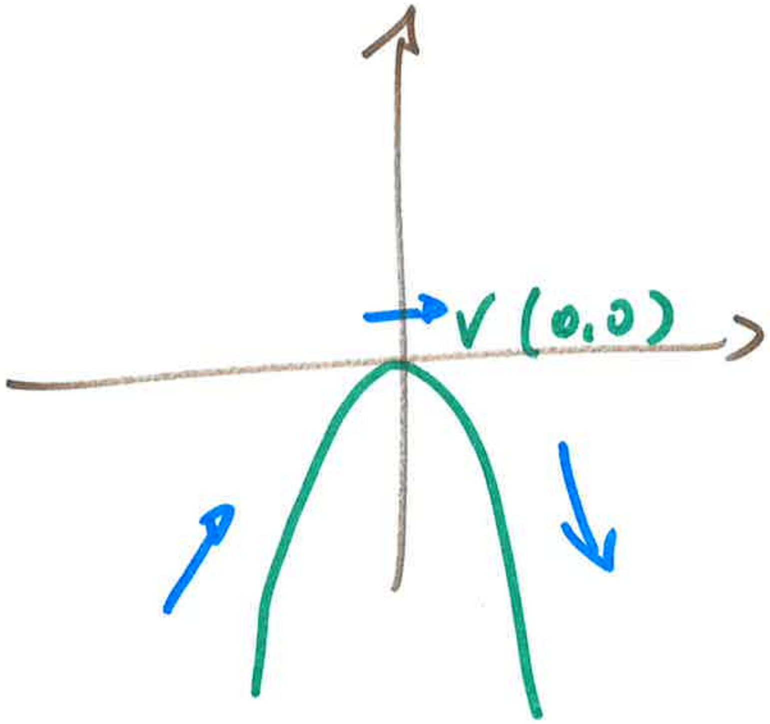
$y' \geq 0$ $\quad 2x \geq 0$

$f(x)$



$x=0$ pt. minimo
(Vertice)

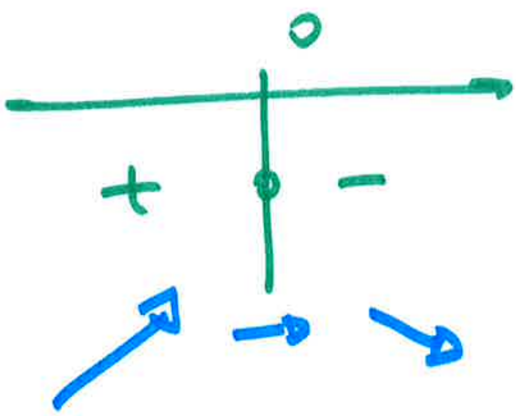
$y = -x^2$ parabola



$y' = -2x$

dom $\mathbb{R}$

$y' \geq 0 \qquad -2x \geq 0$

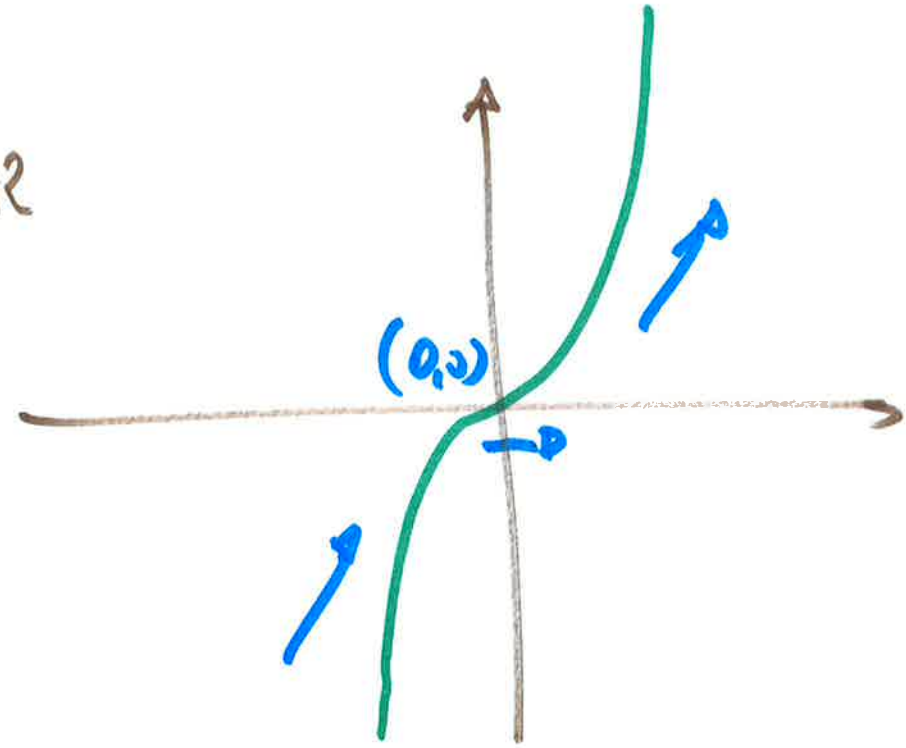


$f(x)$

(0,0) pt. massimo
$x=0$ (vertice)

$y = x^3$ cubica

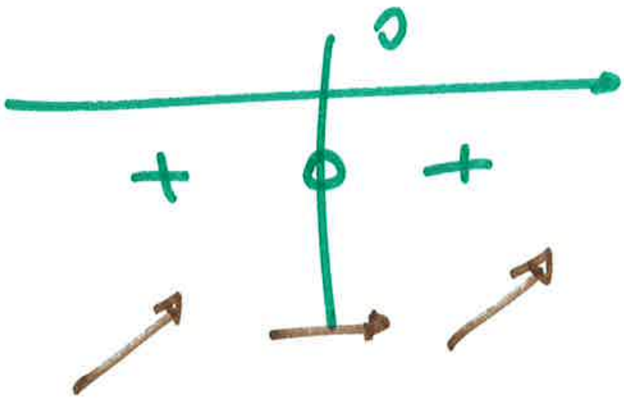
dom $\mathbb{R}$



$y' = 3x^2$

dom $\mathbb{R}$

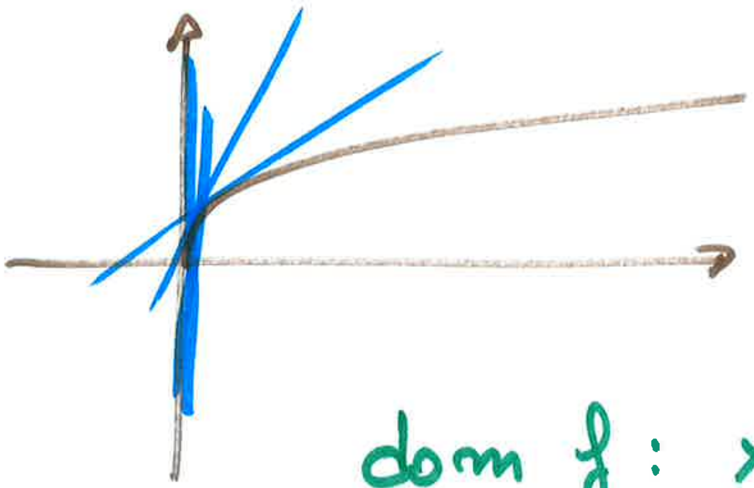
segno: $3x^2 \geq 0$



$y = x^3$

Il punto $(0,0)$ stazionario (perché $y'(0) = 0$), <u>non</u> è di massimo o minimo, pt. tg. orizzontale

$y = \sqrt{x}$ parabola orizzontale



dom f: $x \geq 0$

Calcolare la derivata prima

$y' = \dfrac{1}{2\sqrt{x}}$

dom f': $x > 0$

$x = 0$ f non è derivabile

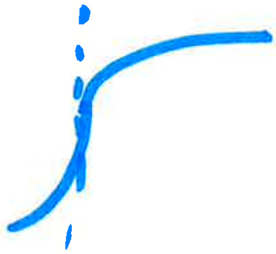
f definita

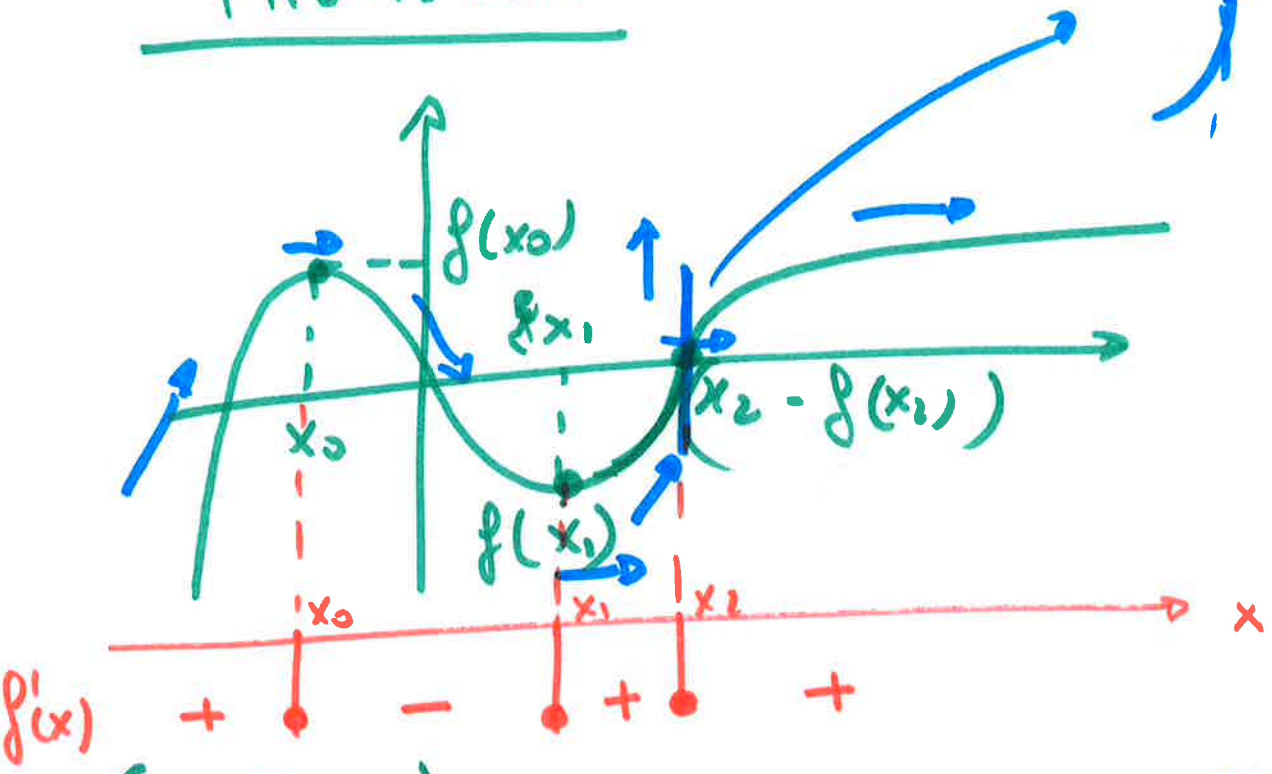
f' non definita

$x = 0$ punto a tangenza verticale

$\left[\lim_{x \to 0^+} y' = +\infty \right]$

PREMESSE

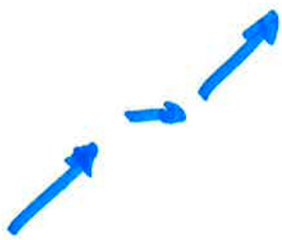


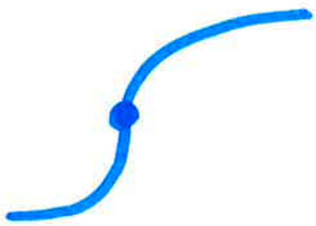


$(x_0\ f(x_0))$ MASSIMO

$(x_1, f(x_1))$ MINIMO

$(x_2, f(x_2))$ Tg. verticale





DERIVATA POSITIVA

FUNZIONE CRESCENTE

DERIVATA NULLA

PT. STAZIONARIO

- massimo
- minimo (Rel. - ass.)
- Tg. verticale
- Tg. orizzontale

DERIVATA NEGATIVA

FUNZIONE DECRESCENTE

/17

• Sia la funzione

$$f(x) = 2 - \sqrt{2}\,\operatorname{arctg}\frac{1}{x} - \sqrt{1+x^2}$$

V - F

(a) $\operatorname{dom} f = \mathbb{R} \setminus \{0\}$

(b) $\lim_{x \to 0^-} f(x) = 1 - \frac{\sqrt{2}}{2}\pi$

(c) $y = x + 2$ asinto obliquo $x \to -\infty$

(d) f è crescente su $]-\infty, 0[$

(e) $x = 1$ è pt. di minimo relativo

V F V ~~F~~ ~~F~~

(a) $x \neq 0$ come arg. di arctg

$\left(1+x^2 \geq 0 \quad \forall x \in \mathbb{R} \text{ come radicando} \right)$

| dom f: $\forall x \in \mathbb{R} \setminus \{0\}$

(b) $\displaystyle \lim_{x \to 0^-} \left(2 - \sqrt{2}\, \operatorname{arctg} \frac{1}{x} - \sqrt{1+x^2} \right) =$

$\frac{1}{x} \rightsquigarrow -\infty$ se $x \to 0^-$

$= 2 - \sqrt{2}\left(-\frac{\pi}{2}\right) - 1 = 1 + \frac{\sqrt{2}}{2}\pi$

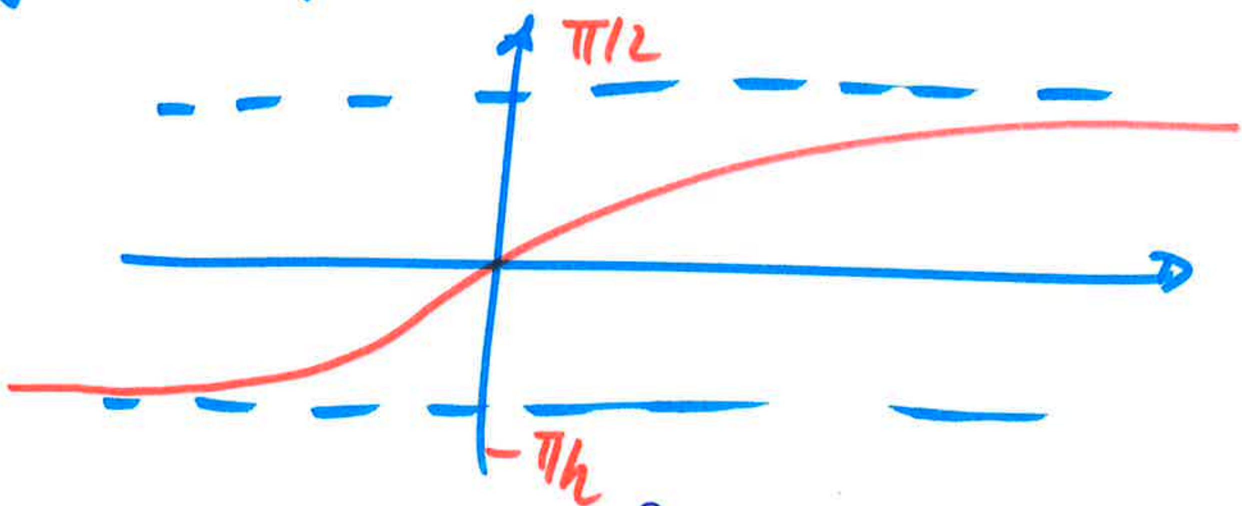
$\displaystyle \lim_{x \to 0^+} \left(2 - \sqrt{2}\, \operatorname{arctg} \frac{1}{x} - \sqrt{1+x^2} \right) =$

$\frac{1}{x} \rightsquigarrow +\infty$ se $x \rightsquigarrow 0^+$

$= 2 - \sqrt{2}\, \frac{\pi}{2} - 1 = 1 - \frac{\sqrt{2}}{2}\pi$

PROMEMORIA

$y = \operatorname{arctg} x$



$$\lim_{x \to \pm\infty} \left(2 - \sqrt{2}\, \operatorname{arctg} \frac{1}{x} - \sqrt{1+x^2}\right) =$$

$\frac{1}{x} \to 0$ per $x \to +\infty$

per $x \to -\infty$

$$= 2 - \sqrt{2} \cdot 0 - \sqrt{1+\infty} = -\infty$$

STUDIO DEL SEGNO

$$2 - \sqrt{2}\, \operatorname{arctg} \frac{1}{x} - \sqrt{1+x^2} \geq 0$$

$$2 - \sqrt{1+x^2} \geq \sqrt{2}\, \operatorname{arctg} \frac{1}{x}$$

$$y = 2 - \sqrt{1+x^2}$$

$$y - 2 = -\sqrt{1+x^2} \qquad y - 2 \leq 0$$

$$(y-2)^2 = 1 + x^2$$

$$x^2 - (y-2)^2 = -1$$

$$x^2 - (y-2)^2 = -1 \quad (-)$$

· iperbole equilatera
Traslata verso l'alto
· verticale

$$x^2 - Y^2 = -1 \quad (-)$$

$$Y = y - 2$$

$$\left\{\begin{array}{l} x = 0 \\ -(y-2)^2 = -1 \end{array}\right.$$

$$(y-2)^2 = 1$$

$$y - 2 = \pm 1$$

$$y = 3 \;/\; y = 1$$

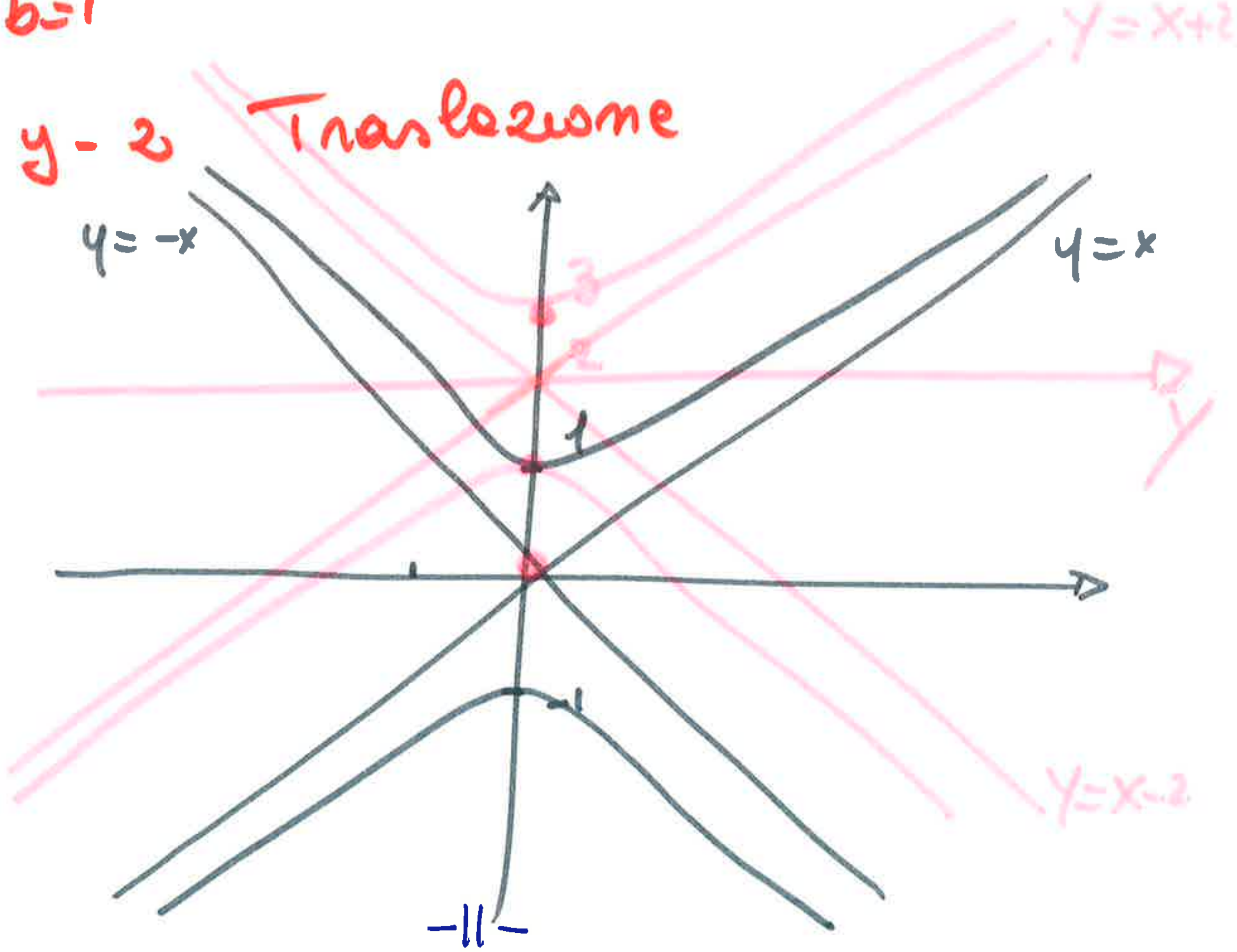
$y = \pm x$ asintoti

(nella Traslazione) Vertici $[0,3]$ $[0,1]$

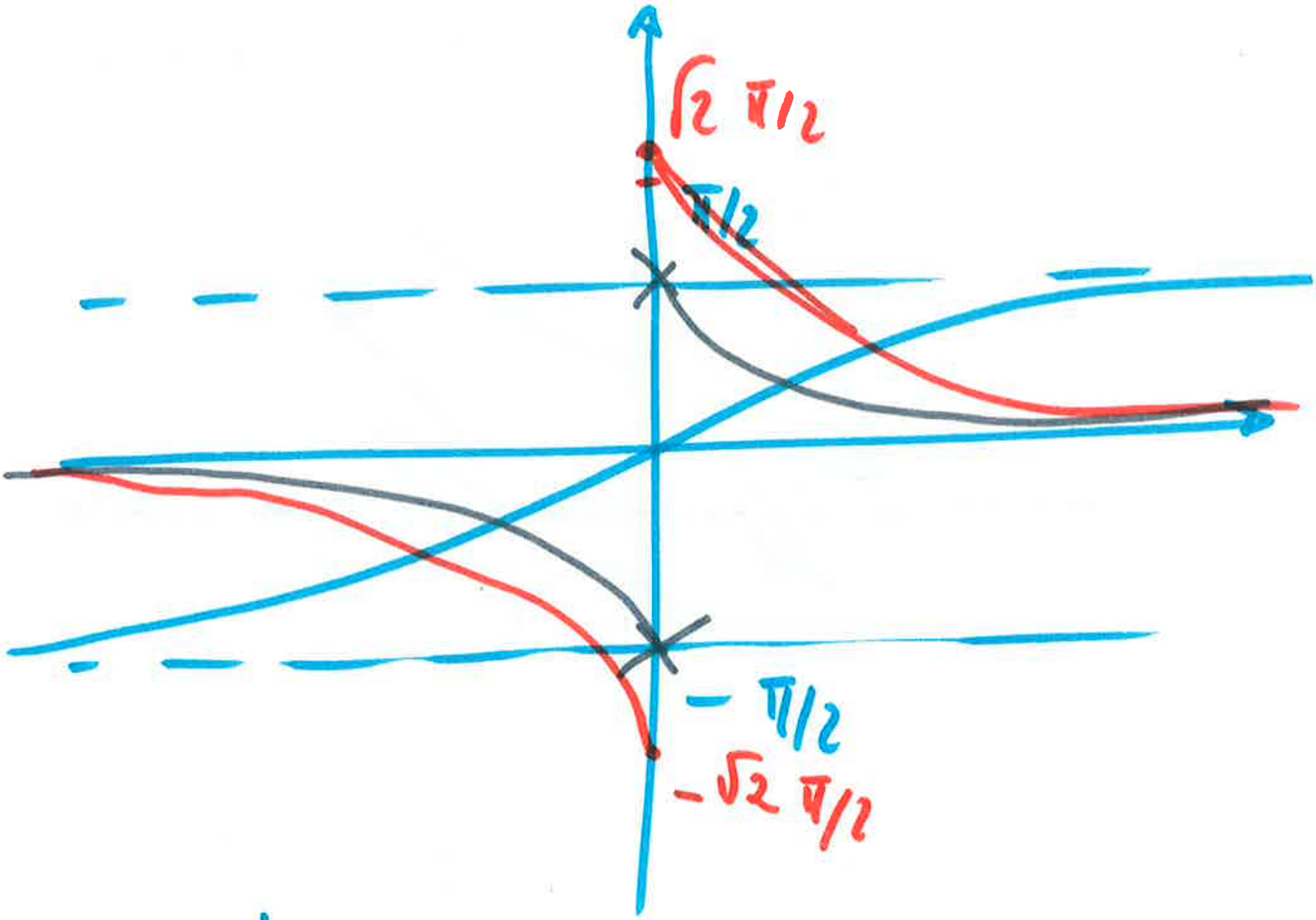
$a = 1$

$b = 1$

$Y = y - 2$ Traslazione



$y = \sqrt{2}\ \text{arctg}\ \frac{1}{x}$

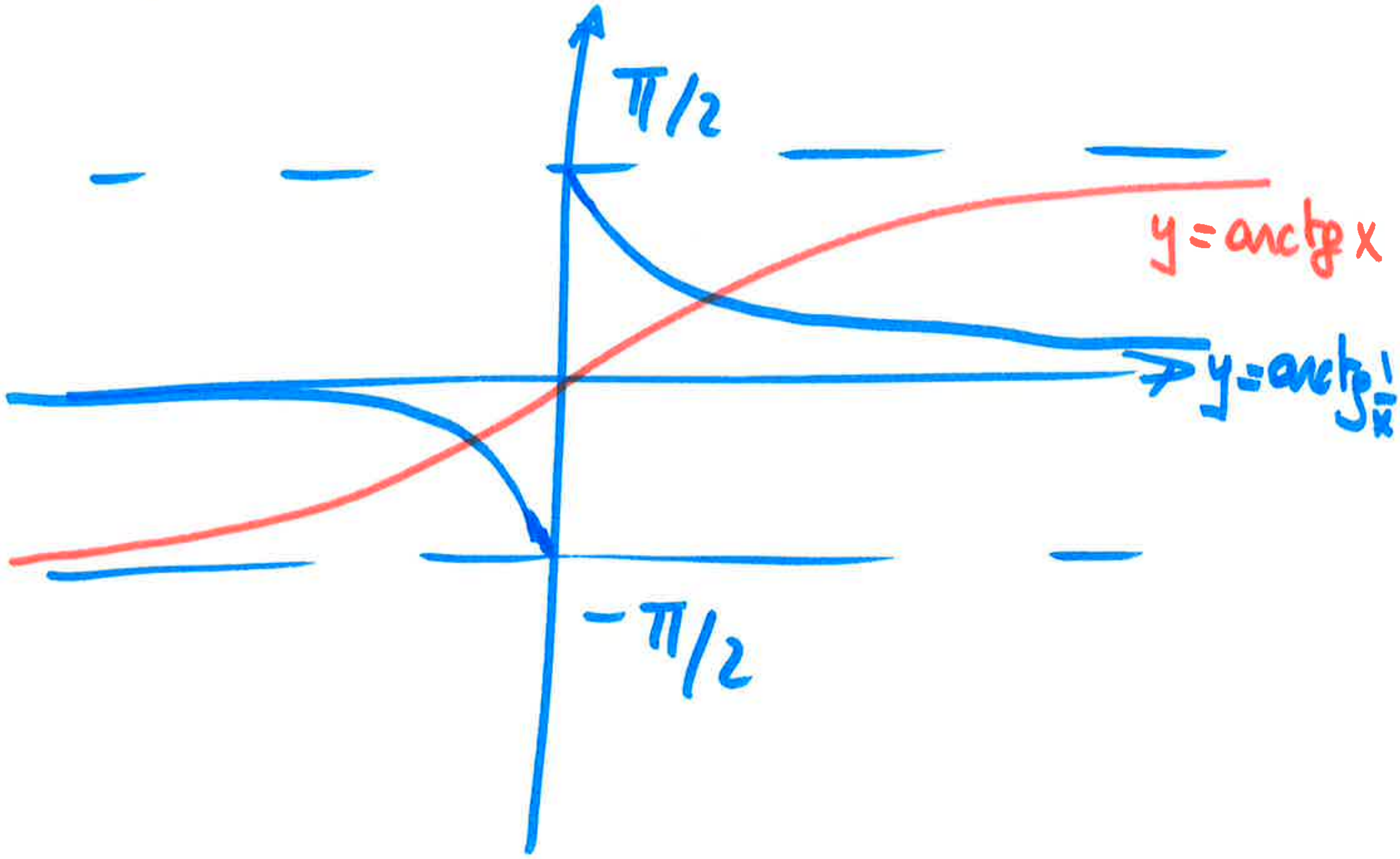


$y = \text{arctg}\ x$

$y = \text{arctg}\ \frac{1}{x}$

$y = \sqrt{2}\ \text{arctg}\ \frac{1}{x}$ (dilatazione di $\sqrt{2}$)

$y = \operatorname{arctg} \frac{1}{x}$



N.B. per lo studio del segno della funzione si devono sovrapporre i 2 grafici ed operare il confronto _

(c)

$$\lim_{x \to -\infty} f(x) = -\infty$$

possibilità di esistenza dell'asintoto obliquo

$$m = \lim_{x \to -\infty} \frac{f(x)}{x} =$$

$$= \lim_{x \to -\infty} \frac{2 - \sqrt{2}\,\operatorname{arctg}\frac{1}{x} - \sqrt{1+x^2}}{x} =$$

$$= \lim_{x \to -\infty} \Bigg[\underbrace{\frac{2}{x}}_{\to 0} - \sqrt{2}\, \underbrace{\frac{\operatorname{arctg}\frac{1}{x}}{x}}_{\to 0} - \frac{\sqrt{1+x^2}}{x} \Bigg]$$

$$\lim_{t \to 0} \frac{\operatorname{arctg} t}{t} = 0$$

$$= \lim_{x \to -\infty} - \frac{\sqrt{x^2(1+1/x^2)}}{x} =$$

$$\lim_{x \to -\infty} \left[-\sqrt{1+x^2} - x \right] =$$

portando fuori $x^2 \to |x|$

sciogliendo il modulo

$$= \lim_{x \to -\infty} \left[x\sqrt{1+\frac{1}{x^2}} - x \right] = 0$$

$\sqrt{1+\frac{1}{x^2}} \to 0$

$$(x - x) = 0$$

$$\left(-\sqrt{1+x^2} - x\right) \cdot \frac{\sqrt{1+x^2} \mp x}{\sqrt{1+x^2} \mp x}$$

Raccogliendo prima il segno

$$-\left(\sqrt{1+x^2} + x\right) \cdot \frac{\sqrt{1+x^2} - x}{\sqrt{1+x^2} - x} =$$

$$= -\frac{1+\cancel{x^2}-\cancel{x^2}}{\sqrt{1+x^2} - x} = -\frac{1}{|x|\sqrt{1+\frac{1}{x^2}} - x} =$$

$|x| = -x$ per $x \to -\infty$

$$= \cancel{-}\frac{1}{\cancel{-}x\left(\sqrt{1+\frac{1}{x^2}} + 1\right)} \to 0$$

$$= \lim_{x \to -\infty} \frac{-|x| \sqrt{1 + 1/x^2}}{x} =$$

$$|x| \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$= \lim_{x \to -\infty} \frac{\cancel{-(-x)} \sqrt{1 + \underbrace{1/x^2}_{\to 0}}}{\cancel{x}} = 1$$

$$m = 1$$

$$q = \lim_{x \to -\infty} \left[f(x) - mx \right] =$$

$$= \lim_{x \to -\infty} \left[2 - \sqrt{2} \underbrace{\operatorname{arctg} \frac{1}{x}}_{0} - \underbrace{\sqrt{1+x^2} - x}_{0} \right] =$$

$$= 2$$

$y = x + 2$ asintoto obliquo sinistro $(x \to -\infty)$

(d) crescente / decrescente

$$y' = -\sqrt{2}\,\frac{1}{1+\frac{1}{x^2}}\cdot\underbrace{\left(-\frac{1}{x^2}\right)}_{D\left(\frac{1}{x}\right)} - \frac{1}{\cancel{2}\sqrt{1+x^2}}\cdot\overbrace{\cancel{2}x}^{D(1+x^2)} =$$

$$= \sqrt{2}\,\frac{1}{\frac{x^2+1}{\cancel{x^2}}}\cdot\frac{1}{\cancel{x^2}} - \frac{x}{\sqrt{1+x^2}} =$$

$$= \frac{\sqrt{2}}{x^2+1} - \frac{x}{\sqrt{1+x^2}} =$$

$$= \frac{\sqrt{2} - x\sqrt{1+x^2}}{x^2+1}$$

$x^2+1 = \left(\sqrt{1+x^2}\right)^2$

$y' \geq 0$

$$\frac{\sqrt{2} - x\sqrt{1+x^2}}{x^2+1} \geq 0 \qquad \text{dom } \forall x \in \mathbb{R}$$

$[\text{dom } f' \supset \text{dom } f]$

den $x^2+1 > 0 \quad \forall x \in \mathbb{R}$

$\Rightarrow$ algebricamente

$$\sqrt{2} - x\sqrt{1+x^2} \geq 0$$

$$\sqrt{2} \geq x\sqrt{1+x^2}$$

COND. OMOG.

$\boxed{x \geq 0}$

$$x^2(1+x^2) \leq 2$$

$$x^2 + x^4 - 2 \leq 0$$

$$x^4 + x^2 - 2 \le 0$$

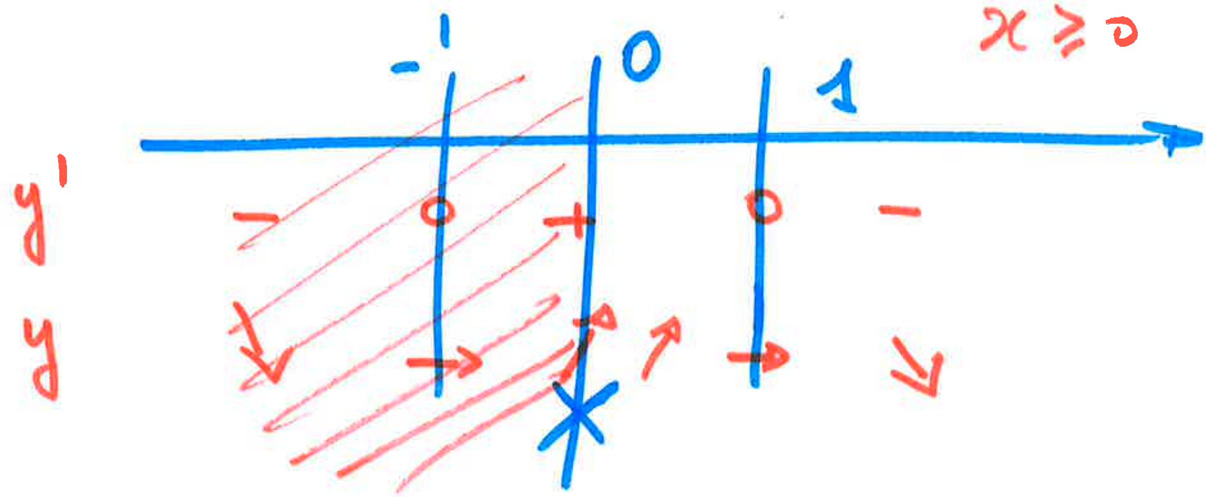
$$\underbrace{(x^2+2)}_{+}\cdot\underbrace{(x^2-1)}_{-} \le 0$$

$$\Downarrow$$

$$x^2-1 \le 0$$

$$-1 \le x \le 1$$

$x \ge 0$



~~$x = -1$ minimo~~

$x = 1$ massimo

$x < 0$

$$\underbrace{x\sqrt{1+x^2}}_{-} \le \sqrt{2}$$

$- \le +$ $\forall x$ sempre vera

$\Rightarrow d$

$\Rightarrow y' \ge 0 \Rightarrow y \nearrow$

$$(x^2+2)(x^2-1) \le 0$$

STUDIO DEL SEGNO

OGNI FATTORE ≥ 0

$x^2+2 \ge 0 \qquad \forall x \in \mathbb{R}$

$x^2-1 \ge 0 \qquad x \le -1 \cup x \ge 1$

		-1		1	
1°	+		+		+
2°	+	0	−	0	+
P.	+	0	−	0	+

$$-1 \le x \le 1$$

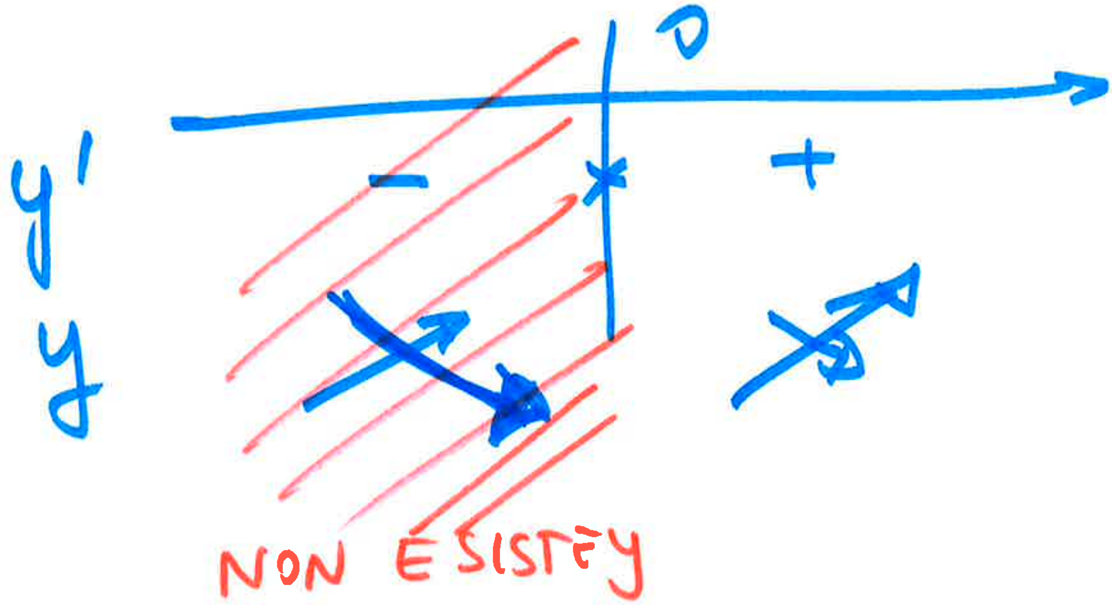
$$y = \log x$$

dom: $x > 0$

$$y' = \frac{1}{x}$$

dom y': $x \neq 0$

dom $y' \supset$ dom y



17

$$f(x) = 2\log|e^x - 1| + \frac{1}{(e^x-1)^2} + 5$$

V-F

(a) $\operatorname{dom} f = \mathbb{R} \setminus \{0, 1\}$

(b) $\lim_{x \to 0^+} f(x) = +\infty$

(c) $y = 2x + 5$ asintoto obliquo $x \to +\infty$

(d) f è decrescente su $]0, \log 2[$

(e) $x = \log 2$ è pt. di massimo relativo

(f) $f^{-1}(]-\infty, 7])$ è un intervallo

F V V V F F

(a)

$$f \quad \frac{|e^x-1|}{e^x-1} \qquad \begin{cases} |e^x-1|>0 \\ e^x-1 \neq 0 \end{cases} \Rightarrow \begin{array}{l} e^x-1\neq 0 \\ e^x \neq 1 = e^0 \\ x \neq 0 \end{array}$$

Falso

(b)

$$\lim_{x\to 0^+} \left\{ 2\log|e^x-1| + \frac{1}{(e^x-1)^2} + 5 \right\} =$$

$$\frac{1}{e^0-1} = \frac{1}{1-1} = \frac{1}{0^+} \to +\infty$$

$$\log|e^0-1| = \log 0^+ = -\infty$$

Forme di indeterminazione

$$= \lim_{x\to 0^+} \left\{ 2\underbrace{\frac{\log|e^x-1|}{e^x}}\cdot e^x + \frac{1}{(e^x-1)^2} + 5 \right\} \simeq$$

Riconducibile al limite notevole $\Rightarrow 1$

$$\simeq \lim_{x\to 0^+} \left\{ 2e^x + \frac{1}{(e^x-1)^2} + 5 \right\} = 0 + \infty + 5 = +\infty$$

Vera

(c) Ricerca asintoto obliquo $x \to +\infty$

• si deve calcolare il limite

$$\lim_{x \to +\infty} \left\{ 2 \cdot \log|e^x - 1| + \frac{1}{(e^x-1)^2} + 5 \right\} = +\infty$$

$$\log|e^{+\infty} - 1| \to +\infty \qquad \frac{1}{\infty} = 0$$

$$m = \lim_{x \to +\infty} \left\{ 2 \cdot \log|e^x - 1| + \frac{1}{(e^x-1)^2} + 5 \right\} \cdot \frac{1}{x} =$$

$$= \lim_{x \to +\infty} \left\{ \frac{2 \cdot \log\left(e^x \left| 1 - \overset{\to 0}{\frac{1}{e^x}} \right| \right)}{x} + \underbrace{\frac{1}{(e^x-1)^2 \cdot x}}_{\to 0} + \underbrace{\frac{5}{x}}_{\to 0} \right\} =$$

$$\simeq \lim_{x \to +\infty} \left\{ 2 \frac{\log(e^x)}{x} \right\} =$$

$$= \lim_{x \to +\infty} \left\{ 2 \frac{\cdot \cancel{x} \cdot \overset{=1}{\log e}}{\cancel{x}} \right\} = 2$$

$$⑨ = \lim_{x\to+\infty} \left\{ 2\log|e^x-1| + \underbrace{\frac{1}{(e^x-1)^2}}_{\downarrow 0} + 5 - 2x \right\} =$$

$$\simeq \lim_{x\to+\infty} \left\{ 2\log e^x \left|1 - \underset{\downarrow 0}{\frac{1}{e^x}}\right| + 5 - 2x \right\} =$$

$$\simeq \lim_{x\to+\infty} \left\{ 2\cdot x \cdot \underbrace{\log e}_{1} + 5 - 2x \right\} = 5$$

Retta asintoto obliquo per $x \to +\infty$ (a destra)

$$y = 2x + 5$$

VERA

(d) crescente / decrescente $\Rightarrow$ derivata 1ª

$$f'(x) = 2\,\frac{1}{|e^x-1|}\cdot e^x \cdot \operatorname{sgn}(e^x-1) + (-2)\cdot(e^x-1)^{-3}\cdot e^x =$$

$$= 2\,\frac{e^x}{|e^x-1|}\operatorname{sgn}(e^x-1) - \frac{2e^x}{(e^x-1)^3}$$

$$f'(x) = \begin{cases} 2\dfrac{e^x}{e^x-1} - \dfrac{2e^x}{(e^x-1)^3} & \text{per } e^x-1 \geq 0 \quad x \geq 0 \\ +2\dfrac{e^x}{+(e^x-1)} - \dfrac{2e^x}{(e^x-1)^3} & \text{per } e^x-1 < 0 \quad x < 0 \end{cases}$$

$$\Rightarrow f'(x) = 2e^x \left[\frac{1}{e^x-1} - \frac{1}{(e^x-1)^3} \right] =$$

$$= 2e^x \frac{(e^x-1)^2 - 1}{(e^x-1)^3}$$

N.B. in questo caso il valore assoluto non modifica il calcolo della derivata prima

$f'(x) \geq 0$ $\qquad\qquad$ dom f' $x \neq 0$

$$2e^x \frac{(e^x-1)^2-1}{(e^x-1)^3} \geq 0$$

N. $(e^x-1)^2 - 1 \geq 0$

$(e^x-1)^2 \geq 1 \Rightarrow e^x - 1 \leq -1 \cup e^x - 1 \geq 1$

$e^x \leq 0$ $\qquad e^x \geq 2$

MAI $\qquad x \geq \log 2$

D. $(e^x-1)^3 > 0$

$e^x - 1 > 0$

$e^x > 1 = e^0 \qquad x > 0$

		0		$\log 2$	
N.	$-$		$-$	$\bullet$	$+$
D.	$-$	$*$	$+$		$+$
F.	$+$	$\times$	$-$	$\bullet$	$+$
$f(x)$	$\nearrow$		$\searrow$	$\rightarrow$	$\nearrow$

$x = \log 2$ è punto di minimo

(e) FALSA **(d) VERA**

(f)

La funzione tra $]-\infty, e7]$

non è invertibile perchè per un tratto è crescente $(-\infty, 0)$, poi decrescente, poi crescente.

(f) FALSA

GRAFICO

$y = f(x)$